OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4753/1

Methods for Advanced Mathematics (C3)

Wednesday **25 MAY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (36 marks)

1 Solve the equation $|3x + 2| = 1$. [3]

2 Given that $\arcsin x = \frac{1}{6}\pi$, find x. Find $\arccos x$ in terms of π. [3]

3 The functions $f(x)$ and $g(x)$ are defined for the domain $x > 0$ as follows:

$$f(x) = \ln x, \quad g(x) = x^3.$$

Express the composite function $fg(x)$ in terms of $\ln x$.

State the transformation which maps the curve $y = f(x)$ onto the curve $y = fg(x)$. [3]

4 The temperature T°C of a liquid at time t minutes is given by the equation

$$T = 30 + 20e^{-0.05t}, \quad \text{for } t \geqslant 0.$$

Write down the initial temperature of the liquid, and find the initial rate of change of temperature.

Find the time at which the temperature is 40 °C. [6]

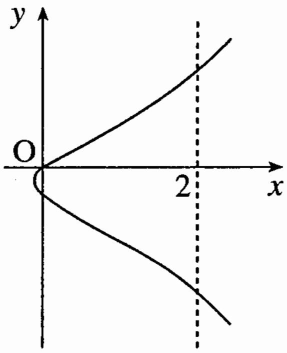
5 Using the substitution $u = 2x + 1$, show that $\int_0^1 \frac{x}{2x+1}\,dx = \frac{1}{4}(2 - \ln 3)$. [6]

6 A curve has equation $y = \frac{x}{2 + 3\ln x}$. Find $\frac{dy}{dx}$. Hence find the exact coordinates of the stationary point of the curve. [7]

7 Fig. 7 shows the curve defined implicitly by the equation

$$y^2 + y = x^3 + 2x,$$

together with the line $x = 2$.



Not to scale

Fig. 7

Find the coordinates of the points of intersection of the line and the curve.

Find $\frac{dy}{dx}$ in terms of x and y. Hence find the gradient of the curve at each of these two points. [8]

Section B (36 marks)

8 Fig. 8 shows part of the curve $y = x \sin 3x$. It crosses the x-axis at P. The point on the curve with x-coordinate $\frac{1}{6}\pi$ is Q.

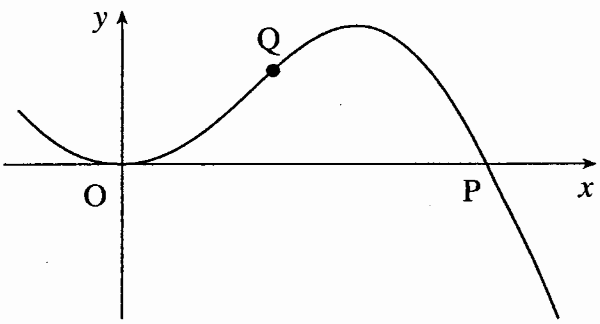


Fig. 8

(i) Find the x-coordinate of P. [3]

(ii) Show that Q lies on the line $y = x$. [1]

(iii) Differentiate $x \sin 3x$. Hence prove that the line $y = x$ touches the curve at Q. [6]

(iv) Show that the area of the region bounded by the curve and the line $y = x$ is $\frac{1}{72}(\pi^2 - 8)$. [7]

[Turn over

9 The function $f(x) = \ln(1 + x^2)$ has domain $-3 \leqslant x \leqslant 3$.

Fig. 9 shows the graph of $y = f(x)$.

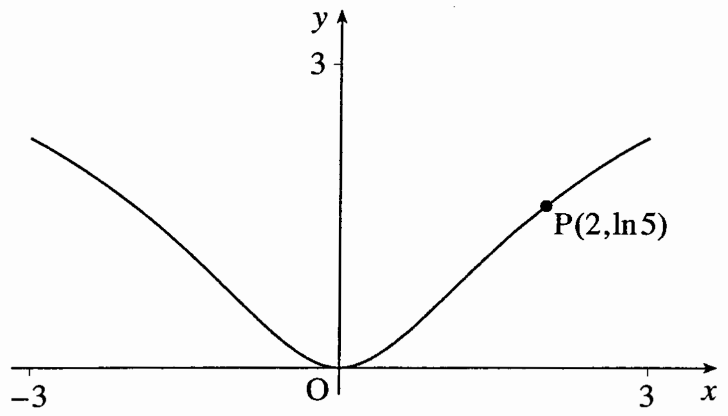


Fig. 9

(i) Show algebraically that the function is even. State how this property relates to the shape of the curve. [3]

(ii) Find the gradient of the curve at the point $P(2, \ln 5)$. [4]

(iii) Explain why the function does not have an inverse for the domain $-3 \leqslant x \leqslant 3$. [1]

The domain of $f(x)$ is now restricted to $0 \leqslant x \leqslant 3$. The inverse of $f(x)$ is the function $g(x)$.

(iv) Sketch the curves $y = f(x)$ and $y = g(x)$ on the same axes.

State the domain of the function $g(x)$.

Show that $g(x) = \sqrt{e^x - 1}$. [6]

(v) Differentiate $g(x)$. Hence verify that $g'(\ln 5) = 1\frac{1}{4}$. Explain the connection between this result and your answer to part **(ii)**. [5]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **4753/1**

Methods for Advanced Mathematics (C3)

Wednesday **18 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is 72.

This question paper consists of 6 printed pages and 2 blank pages.

Section A (36 marks)

1 Given that $y = (1+6x)^{\frac{1}{3}}$, show that $\dfrac{dy}{dx} = \dfrac{2}{y^2}$. [4]

2 A population is P million at time t years. P is modelled by the equation

$$P = 5 + a\mathrm{e}^{-bt},$$

where a and b are constants.

The population is initially 8 million, and declines to 6 million after 1 year.

(i) Use this information to calculate the values of a and b, giving b correct to 3 significant figures. [5]

(ii) What is the long-term population predicted by the model? [1]

3 **(i)** Express $2\ln x + \ln 3$ as a single logarithm. [2]

(ii) Hence, given that x satisfies the equation

$$2\ln x + \ln 3 = \ln(5x+2),$$

show that x is a root of the quadratic equation $3x^2 - 5x - 2 = 0$. [2]

(iii) Solve this quadratic equation, explaining why only one root is a valid solution of

$$2\ln x + \ln 3 = \ln(5x+2).$$ [3]

4 Fig. 4 shows a cone. The angle between the axis and the slant edge is 30°. Water is poured into the cone at a constant rate of 2 cm^3 per second. At time t seconds, the radius of the water surface is r cm and the volume of water in the cone is V cm^3.

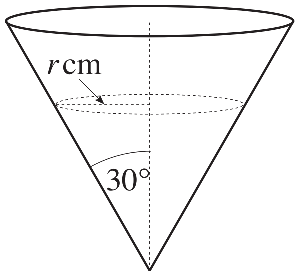


Fig. 4

(i) Write down the value of $\frac{dV}{dt}$. [1]

(ii) Show that $V = \frac{\sqrt{3}}{3}\pi r^3$, and find $\frac{dV}{dr}$. [3]

[You may assume that the volume of a cone of height h and radius r is $\frac{1}{3}\pi r^2 h$.]

(iii) Use the results of parts **(i)** and **(ii)** to find the value of $\frac{dr}{dt}$ when $r = 2$. [3]

5 A curve is defined implicitly by the equation

$$y^3 = 2xy + x^2.$$

(i) Show that $\frac{dy}{dx} = \frac{2(x+y)}{3y^2 - 2x}$. [4]

(ii) Hence write down $\frac{dx}{dy}$ in terms of x and y. [1]

6 The function $f(x)$ is defined by $f(x) = 1 + 2\sin x$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Show that $f^{-1}(x) = \arcsin\left(\frac{x-1}{2}\right)$ and state the domain of this function. [4]

Fig. 6 shows a sketch of the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

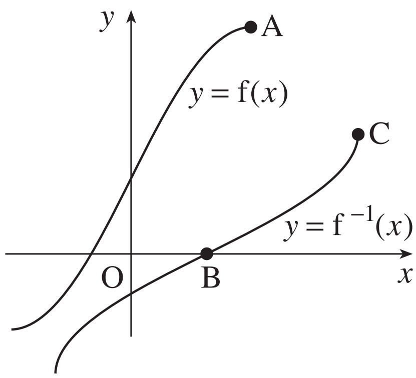


Fig. 6

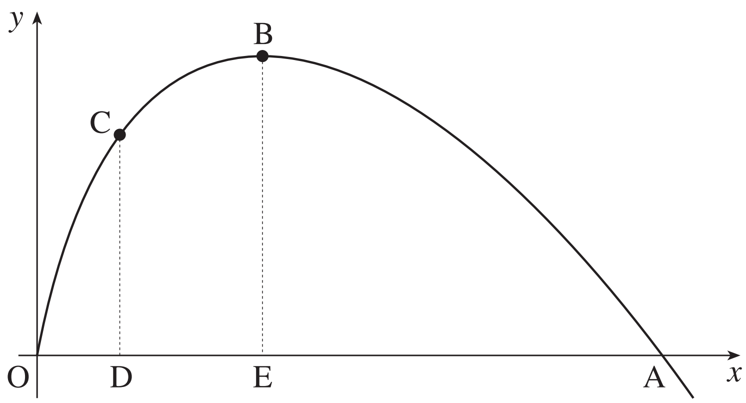
(ii) Write down the coordinates of the points A, B and C. [3]

Section B (36 marks)

7 Fig. 7 shows the curve

$$y = 2x - x \ln x, \text{ where } x > 0.$$

The curve crosses the x-axis at A, and has a turning point at B. The point C on the curve has x-coordinate 1. Lines CD and BE are drawn parallel to the y-axis.



Not to scale

Fig. 7

(i) Find the x-coordinate of A, giving your answer in terms of e. [2]

(ii) Find the exact coordinates of B. [6]

(iii) Show that the tangents at A and C are perpendicular to each other. [3]

(iv) Using integration by parts, show that

$$\int x \ln x \, dx = \tfrac{1}{2}x^2 \ln x - \tfrac{1}{4}x^2 + c.$$

Hence find the exact area of the region enclosed by the curve, the x-axis and the lines CD and BE. [7]

[Question 8 is printed overleaf.]

8 The function $f(x) = \dfrac{\sin x}{2 - \cos x}$ has domain $-\pi \leqslant x \leqslant \pi$.

Fig. 8 shows the graph of $y = f(x)$ for $0 \leqslant x \leqslant \pi$.

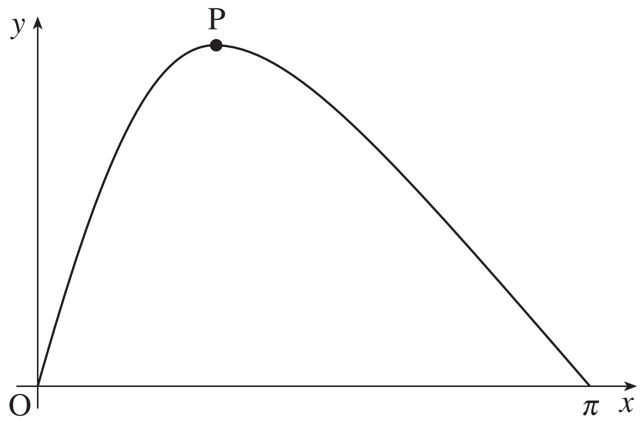


Fig. 8

(i) Find $f(-x)$ in terms of $f(x)$. Hence sketch the graph of $y = f(x)$ for the complete domain $-\pi \leqslant x \leqslant \pi$. [3]

(ii) Show that $f'(x) = \dfrac{2\cos x - 1}{(2 - \cos x)^2}$. Hence find the exact coordinates of the turning point P.

State the range of the function $f(x)$, giving your answer exactly. [8]

(iii) Using the substitution $u = 2 - \cos x$ or otherwise, find the exact value of $\displaystyle\int_0^{\pi} \frac{\sin x}{2 - \cos x}\,dx$. [4]

(iv) Sketch the graph of $y = f(2x)$. [1]

(v) Using your answers to parts **(iii)** and **(iv)**, write down the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{2 - \cos 2x}\,dx$. [2]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4753/1

Methods for Advanced Mathematics (C3)

Thursday **8 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (36 marks)

1 Solve the equation $|3x - 2| = x$. [3]

2 Show that $\displaystyle\int_0^{\frac{1}{6}\pi} x \sin 2x \, dx = \frac{3\sqrt{3} - \pi}{24}$. [6]

3 Fig. 3 shows the curve defined by the equation $y = \arcsin(x - 1)$, for $0 \leqslant x \leqslant 2$.

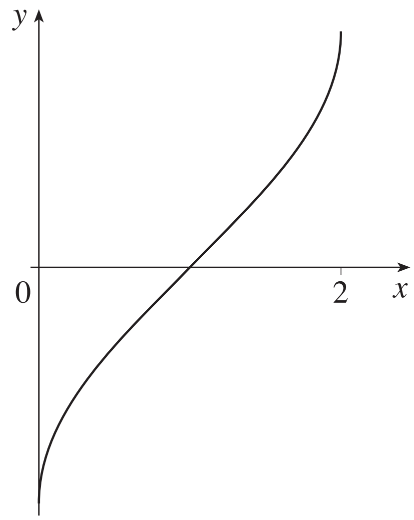

Fig. 3

(i) Find x in terms of y, and show that $\dfrac{dx}{dy} = \cos y$. [3]

(ii) Hence find the exact gradient of the curve at the point where $x = 1.5$. [4]

4 Fig. 4 is a diagram of a garden pond.

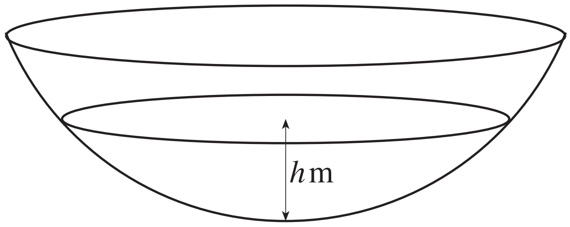

Fig. 4

The volume $V\,\mathrm{m}^3$ of water in the pond when the depth is h metres is given by

$$V = \tfrac{1}{3}\pi h^2(3 - h).$$

(i) Find $\dfrac{\mathrm{d}V}{\mathrm{d}h}$. [2]

Water is poured into the pond at the rate of $0.02\,\mathrm{m}^3$ per minute.

(ii) Find the value of $\dfrac{\mathrm{d}h}{\mathrm{d}t}$ when $h = 0.4$. [4]

5 Positive integers a, b and c are said to form a Pythagorean triple if $a^2 + b^2 = c^2$.

(i) Given that t is an integer greater than 1, show that $2t$, $t^2 - 1$ and $t^2 + 1$ form a Pythagorean triple. [3]

(ii) The two smallest integers of a Pythagorean triple are 20 and 21. Find the third integer.

Use this triple to show that not all Pythagorean triples can be expressed in the form $2t$, $t^2 - 1$ and $t^2 + 1$. [3]

6 The mass M kg of a radioactive material is modelled by the equation

$$M = M_0 \mathrm{e}^{-kt},$$

where M_0 is the initial mass, t is the time in years, and k is a constant which measures the rate of radioactive decay.

(i) Sketch the graph of M against t. [2]

(ii) For Carbon 14, $k = 0.000\,121$. Verify that after 5730 years the mass M has reduced to approximately half the initial mass. [2]

The half-life of a radioactive material is the time taken for its mass to reduce to exactly half the initial mass.

(iii) Show that, in general, the half-life T is given by $T = \dfrac{\ln 2}{k}$. [3]

(iv) Hence find the half-life of Plutonium 239, given that for this material $k = 2.88 \times 10^{-5}$. [1]

[Turn over

Section B (36 marks)

7 Fig. 7 shows the curve $y = \dfrac{x^2 + 3}{x - 1}$. It has a minimum at the point P. The line l is an asymptote to the curve.

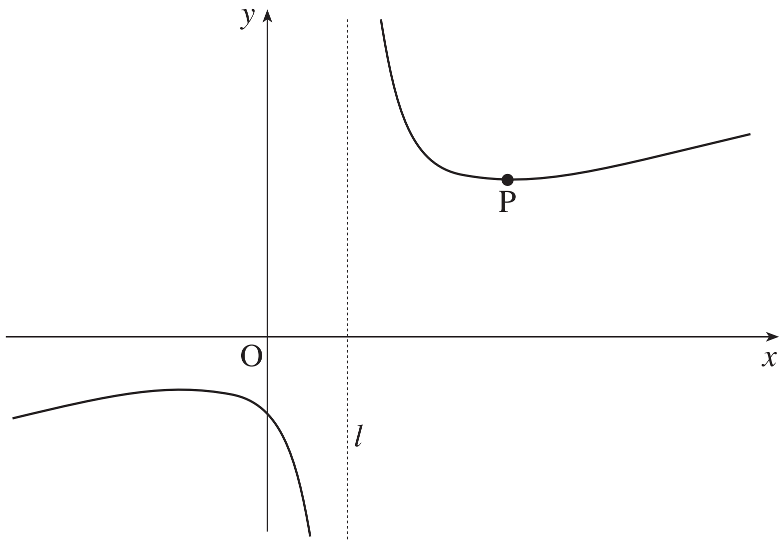

Fig. 7

(i) Write down the equation of the asymptote l. [1]

(ii) Find the coordinates of P. [6]

(iii) Using the substitution $u = x - 1$, show that the area of the region enclosed by the x-axis, the curve and the lines $x = 2$ and $x = 3$ is given by

$$\int_1^2 \left(u + 2 + \frac{4}{u} \right) du.$$

Evaluate this area exactly. [7]

(iv) Another curve is defined by the equation $e^y = \dfrac{x^2 + 3}{x - 1}$. Find $\dfrac{dy}{dx}$ in terms of x and y by differentiating implicitly. Hence find the gradient of this curve at the point where $x = 2$. [4]

8 Fig. 8 shows part of the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \mathrm{e}^{-\frac{1}{5}x}\sin x$, for all x.

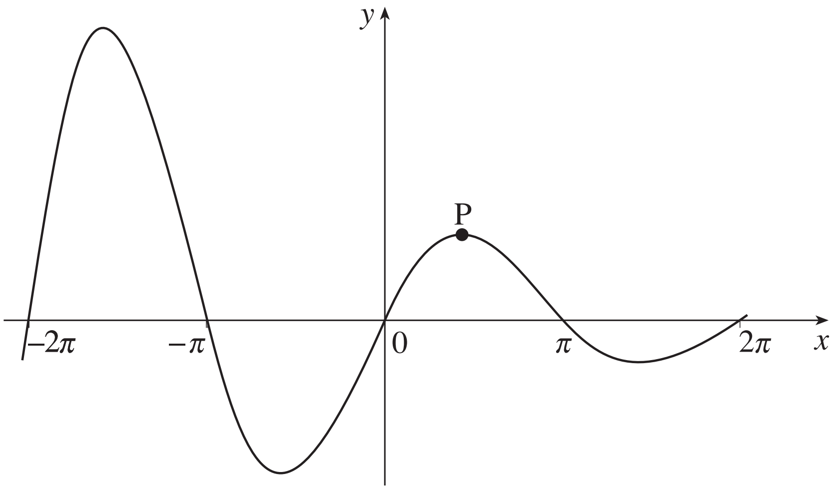

Fig. 8

(i) Sketch the graphs of

(*A*) $y = \mathrm{f}(2x)$,

(*B*) $y = \mathrm{f}(x + \pi)$. [4]

(ii) Show that the x-coordinate of the turning point P satisfies the equation $\tan x = 5$.

Hence find the coordinates of P. [6]

(iii) Show that $\mathrm{f}(x + \pi) = -\mathrm{e}^{-\frac{1}{5}\pi}\mathrm{f}(x)$. Hence, using the substitution $u = x - \pi$, show that

$$\int_{\pi}^{2\pi} \mathrm{f}(x)\,\mathrm{d}x = -\mathrm{e}^{-\frac{1}{5}\pi}\int_{0}^{\pi} \mathrm{f}(u)\,\mathrm{d}u.$$

Interpret this result graphically. [You should *not* attempt to integrate $\mathrm{f}(x)$.] [8]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4753/01**

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

THURSDAY 18 JANUARY 2007

Afternoon
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
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INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
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Section A (36 marks)

1 Fig.1 shows the graphs of $y = |x|$ and $y = |x-2|+1$. The point P is the minimum point of $y = |x-2|+1$, and Q is the point of intersection of the two graphs.

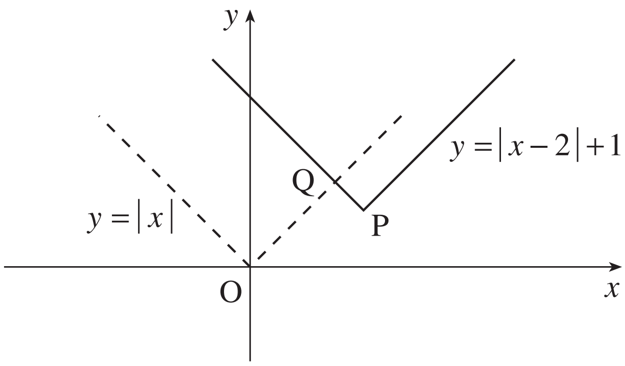

Fig. 1

(i) Write down the coordinates of P. [1]

(ii) Verify that the y-coordinate of Q is $1\frac{1}{2}$. [4]

2 Evaluate $\int_1^2 x^2 \ln x \, dx$, giving your answer in an exact form. [5]

3 The value £V of a car is modelled by the equation $V = Ae^{-kt}$, where t is the age of the car in years and A and k are constants. Its value when new is £10 000, and after 3 years its value is £6000.

(i) Find the values of A and k. [5]

(ii) Find the age of the car when its value is £2000. [2]

4 Use the method of exhaustion to prove the following result.

No 1- or 2-digit perfect square ends in 2, 3, 7 or 8

State a generalisation of this result. [3]

5 The equation of a curve is $y = \dfrac{x^2}{2x+1}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{2x(x+1)}{(2x+1)^2}$. [4]

(ii) Find the coordinates of the stationary points of the curve. You need not determine their nature. [4]

6 Fig. 6 shows the triangle OAP, where O is the origin and A is the point $(0, 3)$. The point $P(x, 0)$ moves on the positive x-axis. The point $Q(0, y)$ moves between O and A in such a way that $AQ + AP = 6$.

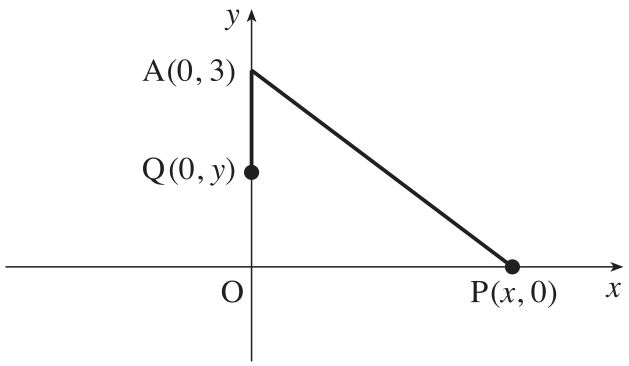

Fig. 6

(i) Write down the length AQ in terms of y. Hence find AP in terms of y, and show that

$$(y+3)^2 = x^2 + 9.$$ [3]

(ii) Use this result to show that $\dfrac{dy}{dx} = \dfrac{x}{y+3}$. [2]

(iii) When $x = 4$ and $y = 2$, $\dfrac{dx}{dt} = 2$. Calculate $\dfrac{dy}{dt}$ at this time. [3]

Section B (36 marks)

7 Fig. 7 shows part of the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x\sqrt{1+x}$. The curve meets the x-axis at the origin and at the point P.

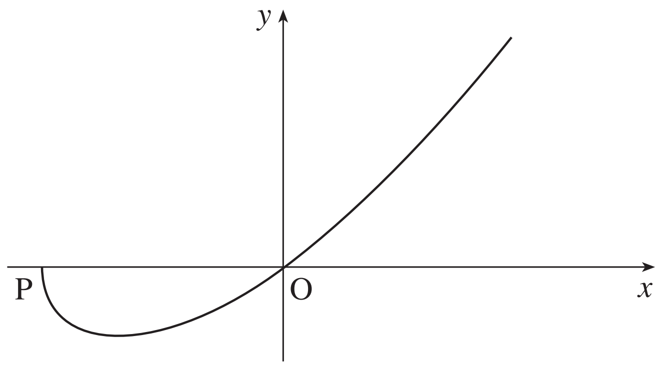

Fig. 7

(i) Verify that the point P has coordinates $(-1, 0)$. Hence state the domain of the function $\mathrm{f}(x)$. [2]

(ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2+3x}{2\sqrt{1+x}}$. [4]

(iii) Find the exact coordinates of the turning point of the curve. Hence write down the range of the function. [4]

(iv) Use the substitution $u = 1 + x$ to show that

$$\int_{-1}^{0} x\sqrt{1+x}\,\mathrm{d}x = \int_{0}^{1}\left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u.$$

Hence find the area of the region enclosed by the curve and the x-axis. [8]

8 Fig. 8 shows part of the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (\mathrm{e}^x - 1)^2 \quad \text{for } x \geqslant 0.$$

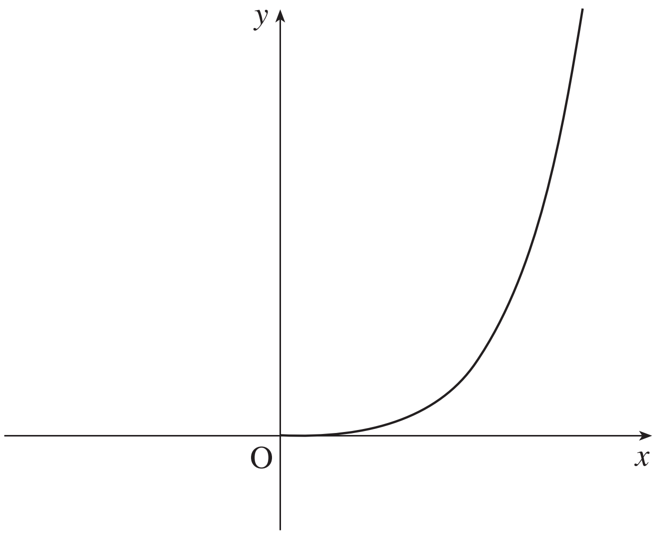

Fig. 8

(i) Find $\mathrm{f}'(x)$, and hence calculate the gradient of the curve $y = \mathrm{f}(x)$ at the origin and at the point $(\ln 2, 1)$. [5]

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \ln\left(1 + \sqrt{x}\right)$ for $x \geqslant 0$.

(ii) Show that $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are inverse functions. Hence sketch the graph of $y = \mathrm{g}(x)$.

Write down the gradient of the curve $y = \mathrm{g}(x)$ at the point $(1, \ln 2)$. [5]

(iii) Show that $\int \left(\mathrm{e}^x - 1\right)^2 \mathrm{d}x = \frac{1}{2}\mathrm{e}^{2x} - 2\mathrm{e}^x + x + c$.

Hence evaluate $\int_0^{\ln 2} \left(\mathrm{e}^x - 1\right)^2 \mathrm{d}x$, giving your answer in an exact form. [5]

(iv) Using your answer to part **(iii)**, calculate the area of the region enclosed by the curve $y = \mathrm{g}(x)$, the x-axis and the line $x = 1$. [3]

RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

MONDAY 11 JUNE 2007

4753/01

Afternoon

Time: 1 hour 30 minutes

Additional materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
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This document consists of **4** printed pages.

Section A (36 marks)

1 **(i)** Differentiate $\sqrt{1+2x}$. [3]

(ii) Show that the derivative of $\ln(1 - e^{-x})$ is $\dfrac{1}{e^x - 1}$. [4]

2 Given that $f(x) = 1 - x$ and $g(x) = |x|$, write down the composite function $gf(x)$.

On separate diagrams, sketch the graphs of $y = f(x)$ and $y = gf(x)$. [3]

3 A curve has equation $2y^2 + y = 9x^2 + 1$.

(i) Find $\dfrac{dy}{dx}$ in terms of x and y. Hence find the gradient of the curve at the point A $(1, 2)$. [4]

(ii) Find the coordinates of the points on the curve at which $\dfrac{dy}{dx} = 0$. [4]

4 A cup of water is cooling. Its initial temperature is 100°C. After 3 minutes, its temperature is 80°C.

(i) Given that $T = 25 + ae^{-kt}$, where T is the temperature in °C, t is the time in minutes and a and k are constants, find the values of a and k. [5]

(ii) What is the temperature of the water

(*A*) after 5 minutes,

(*B*) in the long term? [3]

5 Prove that the following statement is false.

For all integers n greater than or equal to 1, $n^2 + 3n + 1$ is a prime number. [2]

6 Fig. 6 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{1}{2}\arctan x$.

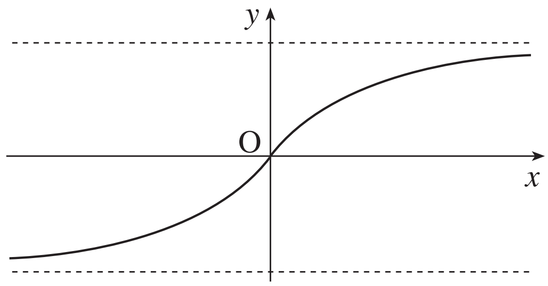

Fig. 6

(i) Find the range of the function $\mathrm{f}(x)$, giving your answer in terms of π. [2]

(ii) Find the inverse function $\mathrm{f}^{-1}(x)$. Find the gradient of the curve $y = \mathrm{f}^{-1}(x)$ at the origin. [5]

(iii) Hence write down the gradient of $y = \frac{1}{2}\arctan x$ at the origin. [1]

Section B (36 marks)

7 Fig. 7 shows the curve $y = \dfrac{x^2}{1 + 2x^3}$. It is undefined at $x = a$; the line $x = a$ is a vertical asymptote.

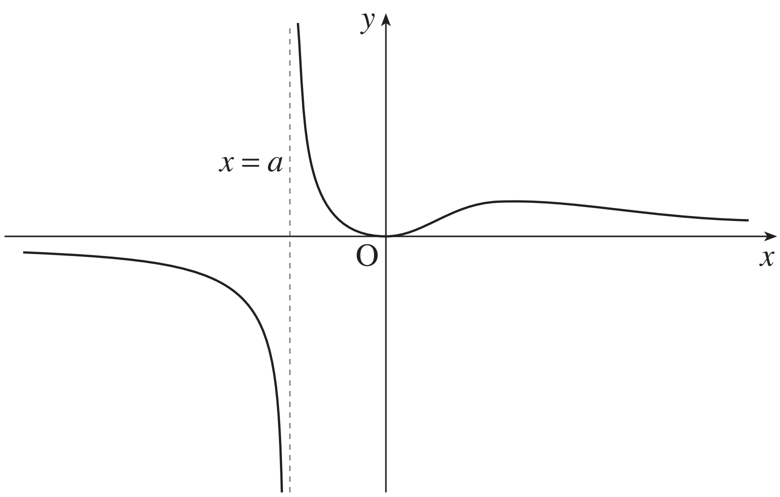

Fig. 7

(i) Calculate the value of a, giving your answer correct to 3 significant figures. [3]

(ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2x - 2x^4}{(1 + 2x^3)^2}$. Hence determine the coordinates of the turning points of the curve. [8]

(iii) Show that the area of the region between the curve and the x-axis from $x = 0$ to $x = 1$ is $\frac{1}{6}\ln 3$. [5]

8 Fig. 8 shows part of the curve $y = x\cos 2x$, together with a point P at which the curve crosses the x-axis.

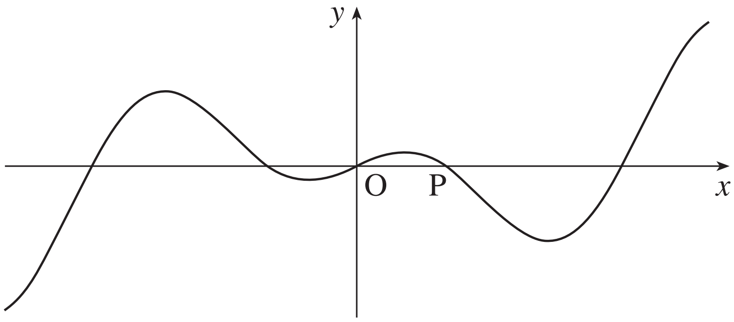

Fig. 8

(i) Find the exact coordinates of P. [3]

(ii) Show algebraically that $x\cos 2x$ is an odd function, and interpret this result graphically. [3]

(iii) Find $\frac{dy}{dx}$. [2]

(iv) Show that turning points occur on the curve for values of x which satisfy the equation $x\tan 2x = \frac{1}{2}$. [2]

(v) Find the gradient of the curve at the origin.

Show that the second derivative of $x\cos 2x$ is zero when $x = 0$. [4]

(vi) Evaluate $\int_0^{\frac{1}{4}\pi} x\cos 2x\,dx$, giving your answer in terms of π. Interpret this result graphically. [6]

RECOGNISING ACHIEVEMENT

ADVANCED GCE **4753/01**

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

MONDAY 2 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
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INFORMATION FOR CANDIDATES

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This document consists of **4** printed pages.

[Turn over

Section A (36 marks)

1 Solve the inequality $|2x - 1| \leqslant 3$. [4]

2 Find $\int x\mathrm{e}^{3x}\,\mathrm{d}x$. [4]

3 **(i)** State the algebraic condition for the function $\mathrm{f}(x)$ to be an even function.

What geometrical property does the graph of an even function have? [2]

(ii) State whether the following functions are odd, even or neither.

(A) $\mathrm{f}(x) = x^2 - 3$

(B) $\mathrm{g}(x) = \sin x + \cos x$

(C) $\mathrm{h}(x) = \dfrac{1}{x + x^3}$ [3]

4 Show that $\displaystyle\int_1^4 \frac{x}{x^2 + 2}\,\mathrm{d}x = \tfrac{1}{2}\ln 6$. [4]

5 Show that the curve $y = x^2 \ln x$ has a stationary point when $x = \dfrac{1}{\sqrt{\mathrm{e}}}$. [6]

6 In a chemical reaction, the mass m grams of a chemical after t minutes is modelled by the equation

$$m = 20 + 30\mathrm{e}^{-0.1t}.$$

(i) Find the initial mass of the chemical.

What is the mass of chemical in the long term? [3]

(ii) Find the time when the mass is 30 grams. [3]

(iii) Sketch the graph of m against t. [2]

7 Given that $x^2 + xy + y^2 = 12$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [5]

Section B (36 marks)

8 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{1+\cos x}$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

P is the point on the curve with x-coordinate $\frac{1}{3}\pi$.

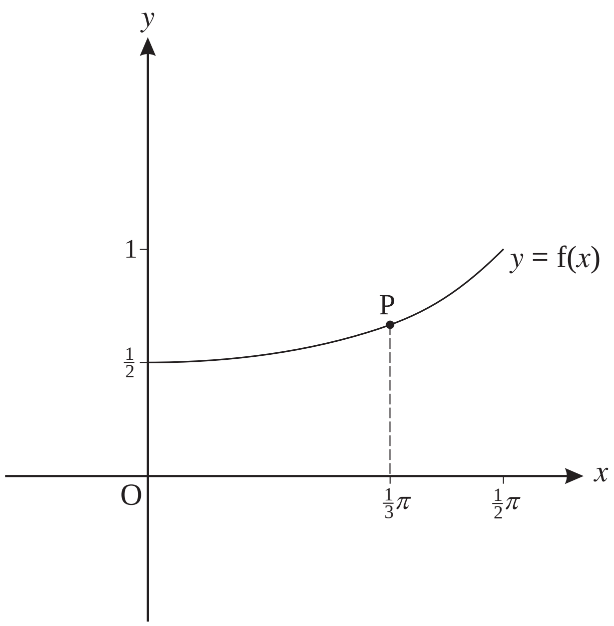

Fig. 8

(i) Find the y-coordinate of P. [1]

(ii) Find $\mathrm{f}'(x)$. Hence find the gradient of the curve at the point P. [5]

(iii) Show that the derivative of $\dfrac{\sin x}{1+\cos x}$ is $\dfrac{1}{1+\cos x}$. Hence find the exact area of the region enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \frac{1}{3}\pi$. [7]

(iv) Show that $\mathrm{f}^{-1}(x) = \arccos\left(\dfrac{1}{x} - 1\right)$. State the domain of this inverse function, and add a sketch of $y = \mathrm{f}^{-1}(x)$ to a copy of Fig. 8. [5]

[Question 9 is printed overleaf.]

9 The function $f(x)$ is defined by $f(x) = \sqrt{4 - x^2}$ for $-2 \leqslant x \leqslant 2$.

(i) Show that the curve $y = \sqrt{4 - x^2}$ is a semicircle of radius 2, and explain why it is not the whole of this circle. [3]

Fig. 9 shows a point P (a, b) on the semicircle. The tangent at P is shown.

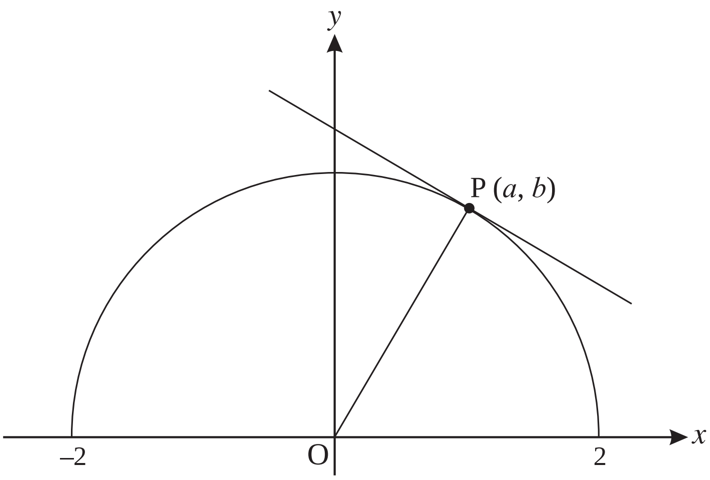

Fig. 9

(ii) (A) Use the gradient of OP to find the gradient of the tangent at P in terms of a and b.

(B) Differentiate $\sqrt{4 - x^2}$ and deduce the value of $f'(a)$.

(C) Show that your answers to parts (A) and (B) are equivalent. [6]

The function $g(x)$ is defined by $g(x) = 3f(x - 2)$, for $0 \leqslant x \leqslant 4$.

(iii) Describe a sequence of two transformations that would map the curve $y = f(x)$ onto the curve $y = g(x)$.

Hence sketch the curve $y = g(x)$. [6]

(iv) Show that if $y = g(x)$ then $9x^2 + y^2 = 36x$. [3]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4753/01**

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

MONDAY 2 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

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 [Turn over

Section A (36 marks)

1 Solve the inequality $|2x - 1| \leqslant 3$. [4]

2 Find $\int x e^{3x} \, dx$. [4]

3 **(i)** State the algebraic condition for the function $f(x)$ to be an even function.

What geometrical property does the graph of an even function have? [2]

(ii) State whether the following functions are odd, even or neither.

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(B) $g(x) = \sin x + \cos x$

(C) $h(x) = \dfrac{1}{x + x^3}$ [3]

4 Show that $\displaystyle\int_1^4 \frac{x}{x^2 + 2} \, dx = \tfrac{1}{2} \ln 6$. [4]

5 Show that the curve $y = x^2 \ln x$ has a stationary point when $x = \dfrac{1}{\sqrt{e}}$. [6]

6 In a chemical reaction, the mass m grams of a chemical after t minutes is modelled by the equation

$$m = 20 + 30e^{-0.1t}.$$

(i) Find the initial mass of the chemical.

What is the mass of chemical in the long term? [3]

(ii) Find the time when the mass is 30 grams. [3]

(iii) Sketch the graph of m against t. [2]

7 Given that $x^2 + xy + y^2 = 12$, find $\dfrac{dy}{dx}$ in terms of x and y. [5]

Section B (36 marks)

8 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{1 + \cos x}$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

P is the point on the curve with x-coordinate $\frac{1}{3}\pi$.

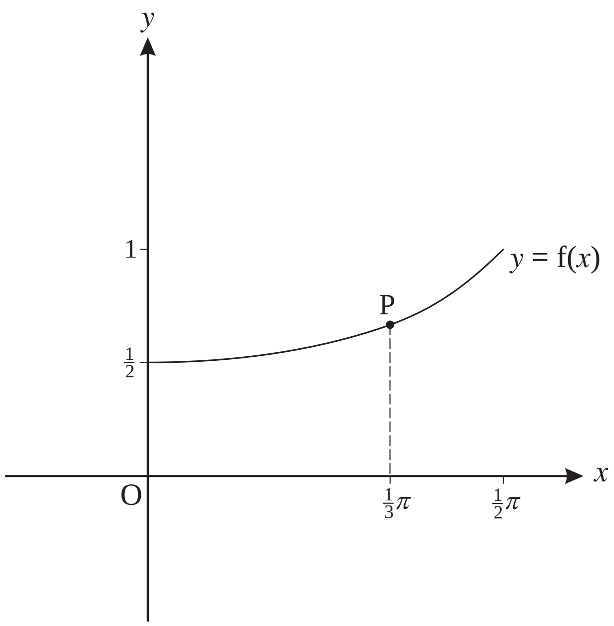

Fig. 8

(i) Find the y-coordinate of P. [1]

(ii) Find $\mathrm{f}'(x)$. Hence find the gradient of the curve at the point P. [5]

(iii) Show that the derivative of $\dfrac{\sin x}{1 + \cos x}$ is $\dfrac{1}{1 + \cos x}$. Hence find the exact area of the region enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \frac{1}{3}\pi$. [7]

(iv) Show that $\mathrm{f}^{-1}(x) = \arccos\left(\dfrac{1}{x} - 1\right)$. State the domain of this inverse function, and add a sketch of $y = \mathrm{f}^{-1}(x)$ to a copy of Fig. 8. [5]

[Question 9 is printed overleaf.]

9 The function $f(x)$ is defined by $f(x) = \sqrt{4 - x^2}$ for $-2 \leqslant x \leqslant 2$.

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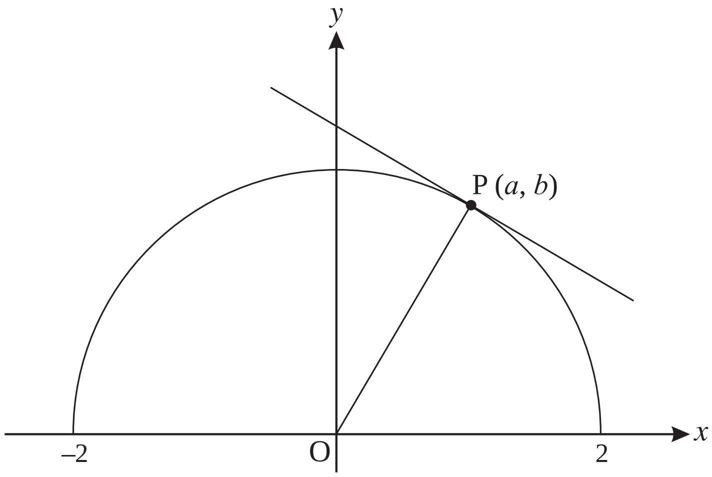

Fig. 9

(ii) (A) Use the gradient of OP to find the gradient of the tangent at P in terms of a and b.

(B) Differentiate $\sqrt{4 - x^2}$ and deduce the value of $f'(a)$.

(C) Show that your answers to parts (A) and (B) are equivalent. [6]

The function $g(x)$ is defined by $g(x) = 3f(x - 2)$, for $0 \leqslant x \leqslant 4$.

(iii) Describe a sequence of two transformations that would map the curve $y = f(x)$ onto the curve $y = g(x)$.

Hence sketch the curve $y = g(x)$. [6]

(iv) Show that if $y = g(x)$ then $9x^2 + y^2 = 36x$. [3]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4753/01**

Methods for Advanced Mathematics (C3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes

4753O1

INSTRUCTIONS TO CANDIDATES

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- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Solve the inequality $|x - 1| < 3$. **[3]**

2 **(i)** Differentiate $x \cos 2x$ with respect to x. **[3]**

(ii) Integrate $x \cos 2x$ with respect to x. **[4]**

3 Given that $\mathrm{f}(x) = \frac{1}{2}\ln(x - 1)$ and $\mathrm{g}(x) = 1 + \mathrm{e}^{2x}$, show that $\mathrm{g}(x)$ is the inverse of $\mathrm{f}(x)$. **[3]**

4 Find the exact value of $\int_0^2 \sqrt{1 + 4x}\,\mathrm{d}x$, showing your working. **[5]**

5 **(i)** State the period of the function $\mathrm{f}(x) = 1 + \cos 2x$, where x is in degrees. **[1]**

(ii) State a sequence of two geometrical transformations which maps the curve $y = \cos x$ onto the curve $y = \mathrm{f}(x)$. **[4]**

(iii) Sketch the graph of $y = \mathrm{f}(x)$ for $-180° < x < 180°$. **[3]**

6 **(i)** Disprove the following statement.

$$\text{‘If } p > q \text{, then } \frac{1}{p} < \frac{1}{q}\text{.’}$$ **[2]**

(ii) State a condition on p and q so that the statement is true. **[1]**

7 The variables x and y satisfy the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$. **[4]**

Both x and y are functions of t.

(ii) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}t}$ when $x = 1$, $y = 8$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 6$. **[3]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = x^2 - \frac{1}{8}\ln x$. P is the point on this curve with x-coordinate 1, and R is the point $\left(0, -\frac{7}{8}\right)$.

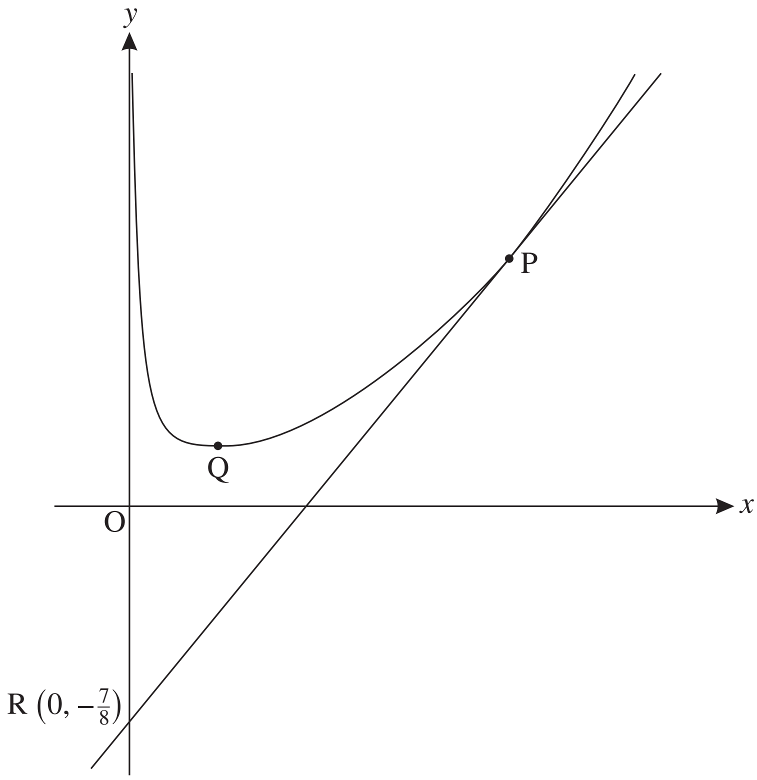

Fig. 8

(i) Find the gradient of PR. **[3]**

(ii) Find $\frac{dy}{dx}$. Hence show that PR is a tangent to the curve. **[3]**

(iii) Find the exact coordinates of the turning point Q. **[5]**

(iv) Differentiate $x \ln x - x$.

Hence, or otherwise, show that the area of the region enclosed by the curve $y = x^2 - \frac{1}{8}\ln x$, the x-axis and the lines $x = 1$ and $x = 2$ is $\frac{59}{24} - \frac{1}{4}\ln 2$. **[7]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{\sqrt{2x - x^2}}$.

The curve has asymptotes $x = 0$ and $x = a$.

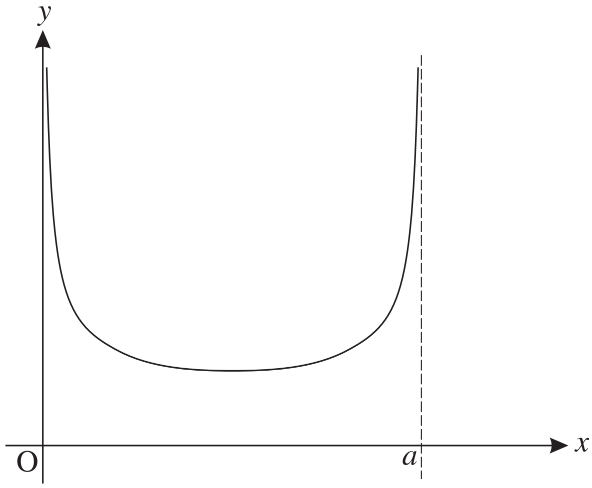

Fig. 9

(i) Find a. Hence write down the domain of the function. **[3]**

(ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x - 1}{(2x - x^2)^{\frac{3}{2}}}$.

Hence find the coordinates of the turning point of the curve, and write down the range of the function. **[8]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \dfrac{1}{\sqrt{1 - x^2}}$.

(iii) (A) Show algebraically that $\mathrm{g}(x)$ is an even function.

(B) Show that $\mathrm{g}(x - 1) = \mathrm{f}(x)$.

(C) Hence prove that the curve $y = \mathrm{f}(x)$ is symmetrical, and state its line of symmetry. **[7]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4753/01**

Methods for Advanced Mathematics (C3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes

* 4 7 5 3 0 1 *

INSTRUCTIONS TO CANDIDATES

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Section A (36 marks)

1 Evaluate $\int_0^{\frac{1}{6}\pi} \sin 3x \, dx$. **[3]**

2 A radioactive substance decays exponentially, so that its mass M grams can be modelled by the equation $M = Ae^{-kt}$, where t is the time in years, and A and k are positive constants.

(i) An initial mass of 100 grams of the substance decays to 50 grams in 1500 years. Find A and k. **[5]**

(ii) The substance becomes safe when 99% of its initial mass has decayed. Find how long it will take before the substance becomes safe. **[3]**

3 Sketch the curve $y = 2 \arccos x$ for $-1 \leqslant x \leqslant 1$. **[3]**

4 Fig. 4 shows a sketch of the graph of $y = 2|x - 1|$. It meets the x- and y-axes at $(a, 0)$ and $(0, b)$ respectively.

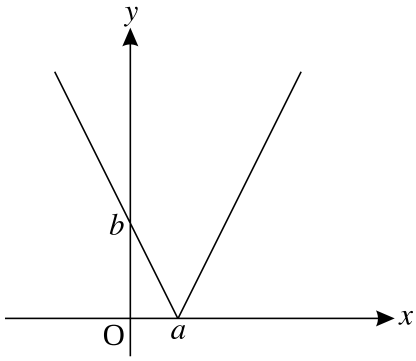

Fig. 4

Find the values of a and b. **[3]**

5 The equation of a curve is given by $e^{2y} = 1 + \sin x$.

(i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y. **[3]**

(ii) Find an expression for y in terms of x, and differentiate it to verify the result in part **(i)**. **[4]**

6 Given that $f(x) = \frac{x+1}{x-1}$, show that $ff(x) = x$.

Hence write down the inverse function $f^{-1}(x)$. What can you deduce about the symmetry of the curve $y = f(x)$? **[5]**

7 **(i)** Show that

(A) $(x - y)(x^2 + xy + y^2) = x^3 - y^3$,

(B) $(x + \frac{1}{2}y)^2 + \frac{3}{4}y^2 = x^2 + xy + y^2$. **[4]**

(ii) Hence prove that, for all real numbers x and y, if $x > y$ then $x^3 > y^3$. **[3]**

Section B (36 marks)

8 Fig. 8 shows the line $y = x$ and parts of the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, where

$$\mathrm{f}(x) = \mathrm{e}^{x-1}, \quad \mathrm{g}(x) = 1 + \ln x.$$

The curves intersect the axes at the points A and B, as shown. The curves and the line $y = x$ meet at the point C.

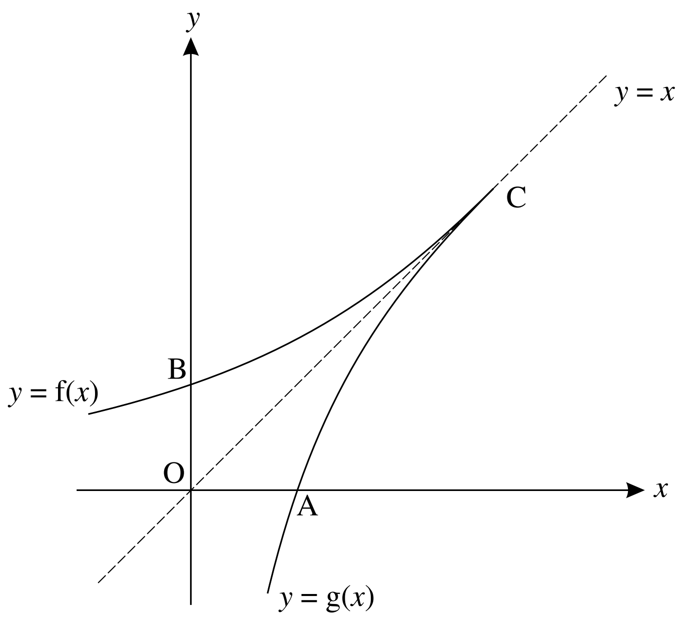

Fig. 8

(i) Find the exact coordinates of A and B. Verify that the coordinates of C are $(1, 1)$. **[5]**

(ii) Prove algebraically that $\mathrm{g}(x)$ is the inverse of $\mathrm{f}(x)$. **[2]**

(iii) Evaluate $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in terms of e. **[3]**

(iv) Use integration by parts to find $\int \ln x\,\mathrm{d}x$.

Hence show that $\int_{\mathrm{e}^{-1}}^{1} \mathrm{g}(x)\,\mathrm{d}x = \frac{1}{\mathrm{e}}$. **[6]**

(v) Find the area of the region enclosed by the lines OA and OB, and the arcs AC and BC. **[2]**

9 Fig. 9 shows the curve $y = \dfrac{x^2}{3x-1}$.

P is a turning point, and the curve has a vertical asymptote $x = a$.

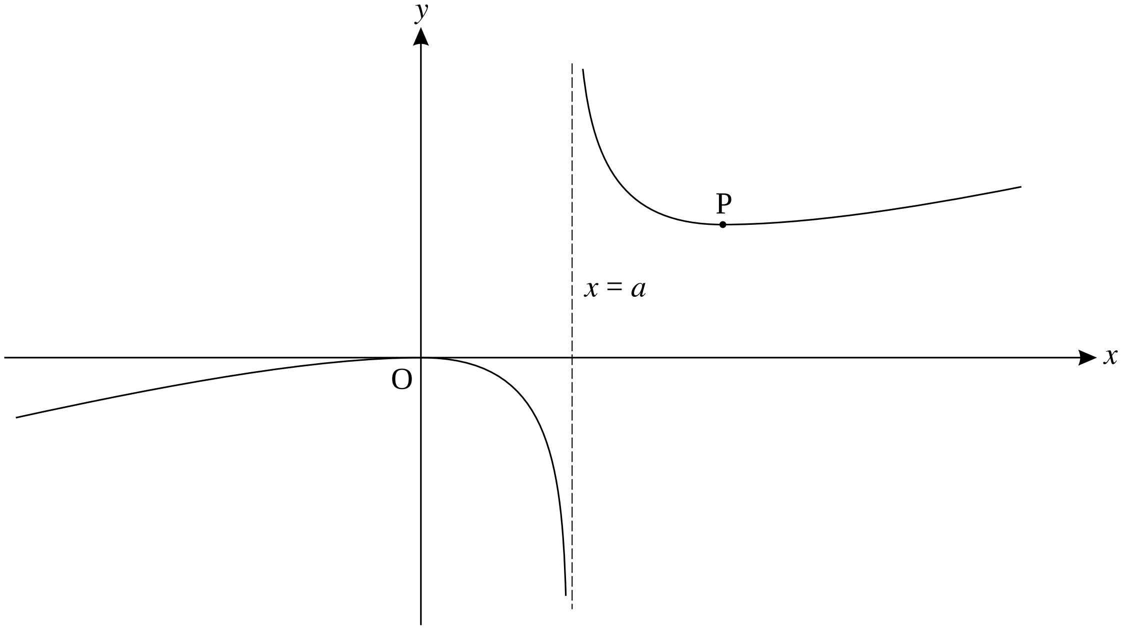

Fig. 9

(i) Write down the value of a. **[1]**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{x(3x-2)}{(3x-1)^2}$. **[3]**

(iii) Find the exact coordinates of the turning point P.

Calculate the gradient of the curve when $x = 0.6$ and $x = 0.8$, and hence verify that P is a minimum point. **[7]**

(iv) Using the substitution $u = 3x - 1$, show that $\displaystyle\int \frac{x^2}{3x-1}\,dx = \frac{1}{27}\int \left(u + 2 + \frac{1}{u}\right) du$.

Hence find the exact area of the region enclosed by the curve, the x-axis and the lines $x = \frac{2}{3}$ and $x = 1$. **[7]**

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RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4753/01**

Methods for Advanced Mathematics (C3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

4753010110

INSTRUCTIONS TO CANDIDATES

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Section A (36 marks)

1 Solve the equation $e^{2x} - 5e^{x} = 0$. **[4]**

2 The temperature T in degrees Celsius of water in a glass t minutes after boiling is modelled by the equation $T = 20 + be^{-kt}$, where b and k are constants. Initially the temperature is $100\,^{\circ}\mathrm{C}$, and after 5 minutes the temperature is $60\,^{\circ}\mathrm{C}$.

(i) Find b and k. **[4]**

(ii) Find at what time the temperature reaches $50\,^{\circ}\mathrm{C}$. **[2]**

3 **(i)** Given that $y = \sqrt[3]{1 + 3x^2}$, use the chain rule to find $\dfrac{dy}{dx}$ in terms of x. **[3]**

(ii) Given that $y^3 = 1 + 3x^2$, use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of x and y. Show that this result is equivalent to the result in part **(i)**. **[4]**

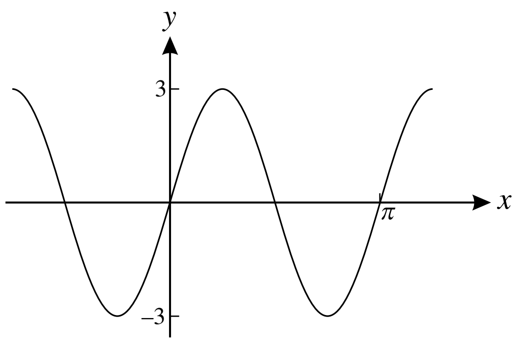
4 Evaluate the following integrals, giving your answers in exact form.

(i) $\displaystyle\int_0^1 \frac{2x}{x^2 + 1}\,dx$. **[3]**

(ii) $\displaystyle\int_0^1 \frac{2x}{x + 1}\,dx$. **[5]**

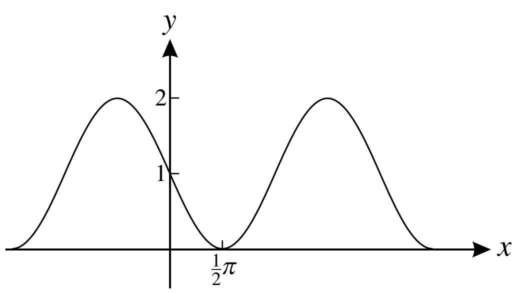
5 The curves in parts **(i)** and **(ii)** have equations of the form $y = a + b\sin cx$, where a, b and c are constants. For each curve, find the values of a, b and c.

(i)



[2]

(ii)



[2]

6 Write down the conditions for $f(x)$ to be an odd function and for $g(x)$ to be an even function.

Hence prove that, if $f(x)$ is odd and $g(x)$ is even, then the composite function $gf(x)$ is even. **[4]**

7 Given that $\arcsin x = \arccos y$, prove that $x^2 + y^2 = 1$. [Hint: let $\arcsin x = \theta$.] **[3]**

Section B (36 marks)

8 Fig. 8 shows part of the curve $y = x \cos 3x$.

The curve crosses the x-axis at O, P and Q.

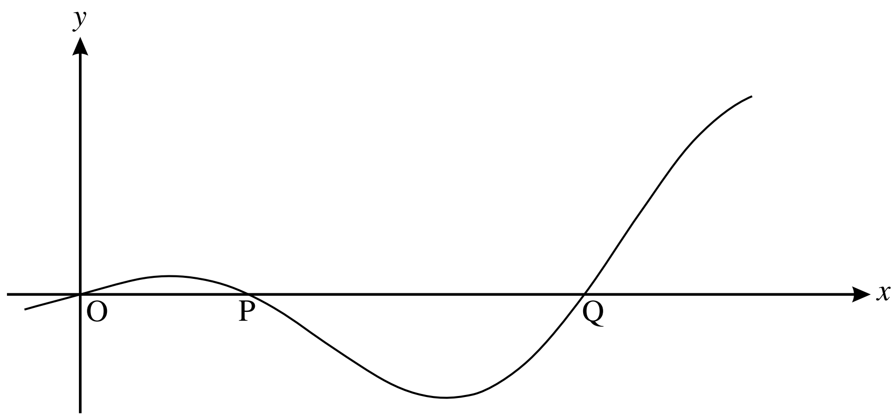


Fig. 8

(i) Find the exact coordinates of P and Q. **[4]**

(ii) Find the exact gradient of the curve at the point P.

Show also that the turning points of the curve occur when $x \tan 3x = \frac{1}{3}$. **[7]**

(iii) Find the area of the region enclosed by the curve and the x-axis between O and P, giving your answer in exact form. **[6]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{2x^2 - 1}{x^2 + 1}$ for the domain $0 \leqslant x \leqslant 2$.

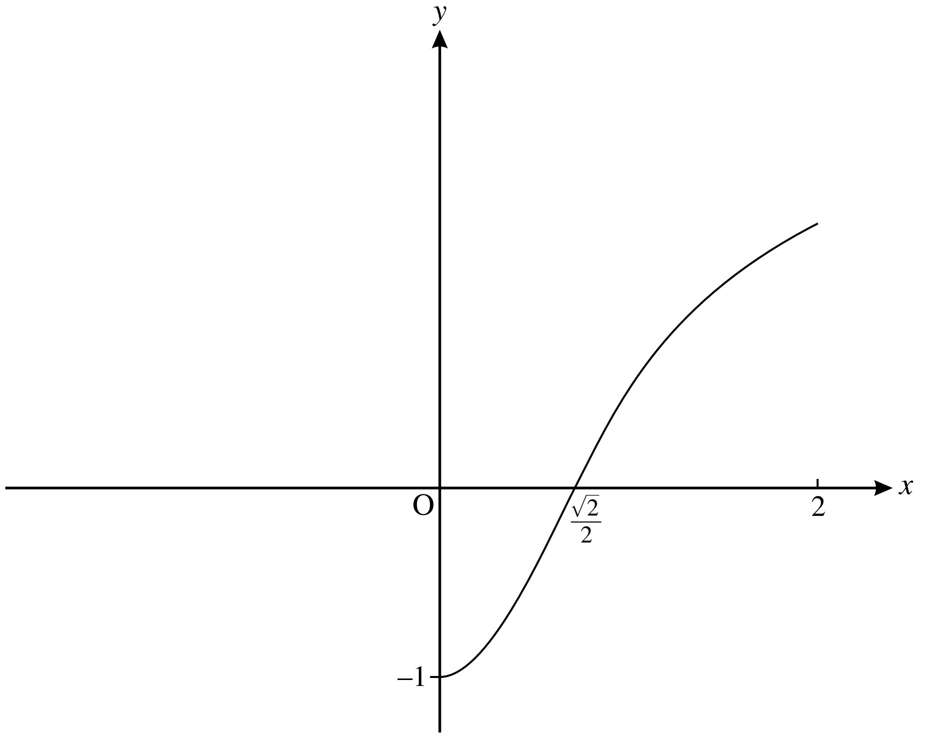


Fig. 9

(i) Show that $\mathrm{f}'(x) = \dfrac{6x}{(x^2 + 1)^2}$, and hence that $\mathrm{f}(x)$ is an increasing function for $x > 0$. **[5]**

(ii) Find the range of $\mathrm{f}(x)$. **[2]**

(iii) **Given** that $\mathrm{f}''(x) = \dfrac{6 - 18x^2}{(x^2 + 1)^3}$, find the maximum value of $\mathrm{f}'(x)$. **[4]**

The function $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

(iv) Write down the domain and range of $\mathrm{g}(x)$. Add a sketch of the curve $y = \mathrm{g}(x)$ to a copy of Fig. 9. **[4]**

(v) Show that $\mathrm{g}(x) = \sqrt{\dfrac{x + 1}{2 - x}}$. **[4]**

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ADVANCED GCE

MATHEMATICS (MEI) **4753/01**

Methods for Advanced Mathematics (C3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Friday 11 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Evaluate $\int_0^{\frac{1}{6}\pi} \cos 3x \, dx$. **[3]**

2 Given that $f(x) = |x|$ and $g(x) = x + 1$, sketch the graphs of the composite functions $y = fg(x)$ and $y = gf(x)$, indicating clearly which is which. **[4]**

3 **(i)** Differentiate $\sqrt{1 + 3x^2}$. **[3]**

(ii) Hence show that the derivative of $x\sqrt{1 + 3x^2}$ is $\dfrac{1 + 6x^2}{\sqrt{1 + 3x^2}}$. **[4]**

4 A piston can slide inside a tube which is closed at one end and encloses a quantity of gas (see Fig. 4).

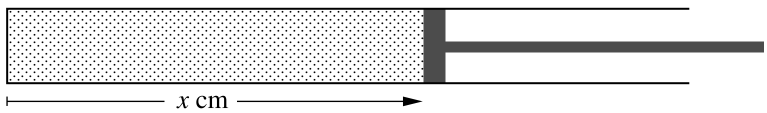

Fig. 4

The pressure of the gas in atmospheric units is given by $p = \dfrac{100}{x}$, where x cm is the distance of the piston from the closed end. At a certain moment, $x = 50$, and the piston is being pulled away from the closed end at 10 cm per minute. At what rate is the pressure changing at that time? **[6]**

5 Given that $y^3 = xy - x^2$, show that $\dfrac{dy}{dx} = \dfrac{y - 2x}{3y^2 - x}$.

Hence show that the curve $y^3 = xy - x^2$ has a stationary point when $x = \frac{1}{8}$. **[7]**

6 The function $f(x)$ is defined by

$$f(x) = 1 + 2\sin 3x, \quad -\frac{\pi}{6} \leqslant x \leqslant \frac{\pi}{6}.$$

You are given that this function has an inverse, $f^{-1}(x)$.

Find $f^{-1}(x)$ and its domain. **[6]**

7 State whether the following statements are true or false; if false, provide a counter-example.

(i) If a is rational and b is rational, then $a + b$ is rational.

(ii) If a is rational and b is irrational, then $a + b$ is irrational.

(iii) If a is irrational and b is irrational, then $a + b$ is irrational. **[3]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = 3\ln x + x - x^2$.

The curve crosses the x-axis at P and Q, and has a turning point at R. The x-coordinate of Q is approximately 2.05.

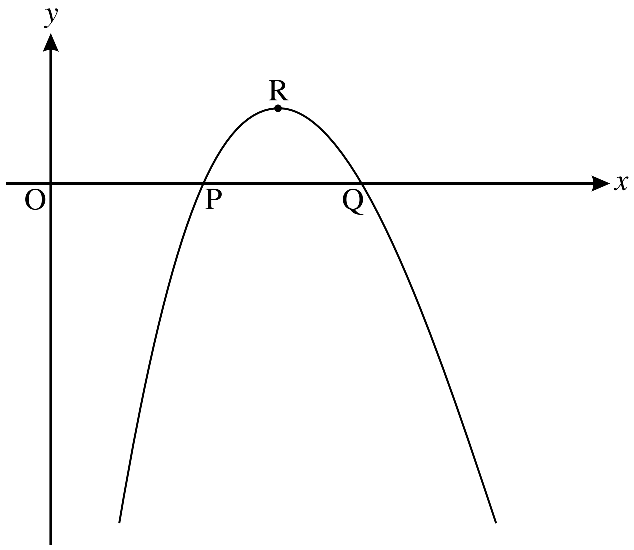

Fig. 8

(i) Verify that the coordinates of P are (1, 0). **[1]**

(ii) Find the coordinates of R, giving the y-coordinate correct to 3 significant figures.

Find $\frac{d^2y}{dx^2}$, and use this to verify that R is a maximum point. **[9]**

(iii) Find $\int \ln x \, dx$.

Hence calculate the area of the region enclosed by the curve and the x-axis between P and Q, giving your answer to 2 significant figures. **[7]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{\mathrm{e}^{2x}}{1 + \mathrm{e}^{2x}}$. The curve crosses the y-axis at P.

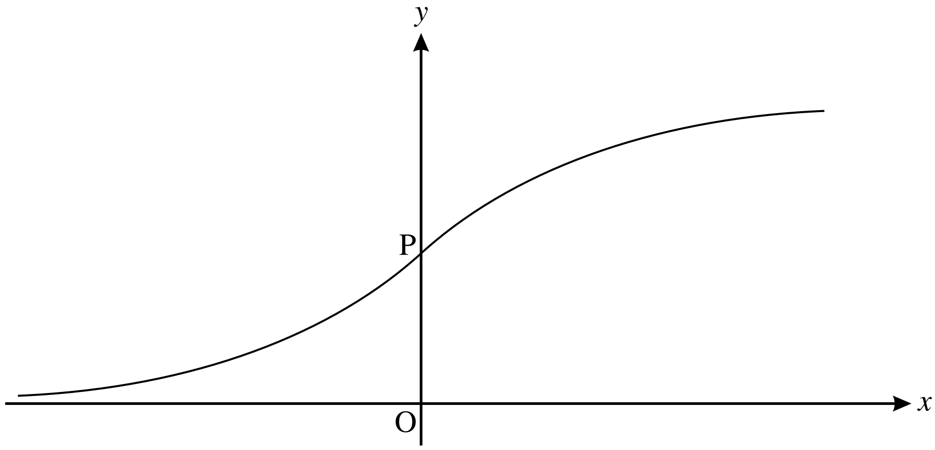

Fig. 9

(i) Find the coordinates of P. **[1]**

(ii) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, simplifying your answer.

Hence calculate the gradient of the curve at P. **[4]**

(iii) Show that the area of the region enclosed by $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = 1$ is $\frac{1}{2}\ln\left(\dfrac{1 + \mathrm{e}^2}{2}\right)$. **[5]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \frac{1}{2}\left(\dfrac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}\right)$.

(iv) Prove algebraically that $\mathrm{g}(x)$ is an odd function.

Interpret this result graphically. **[3]**

(v) (*A*) Show that $\mathrm{g}(x) + \frac{1}{2} = \mathrm{f}(x)$.

(*B*) Describe the transformation which maps the curve $y = \mathrm{g}(x)$ onto the curve $y = \mathrm{f}(x)$.

(*C*) What can you conclude about the symmetry of the curve $y = \mathrm{f}(x)$? **[6]**

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ADVANCED GCE

MATHEMATICS (MEI) **4753/01**

Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 19 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 Given that $y = \sqrt[3]{1 + x^2}$, find $\dfrac{dy}{dx}$. **[4]**

2 Solve the inequality $|2x + 1| \geqslant 4$. **[4]**

3 The area of a circular stain is growing at a rate of 1 mm^2 per second. Find the rate of increase of its radius at an instant when its radius is 2 mm. **[5]**

4 Use the triangle in Fig. 4 to prove that $\sin^2\theta + \cos^2\theta = 1$. For what values of θ is this proof valid? **[3]**

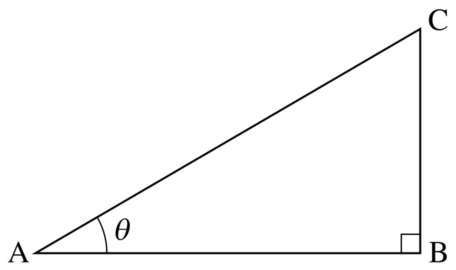

Fig. 4

5 **(i)** On a single set of axes, sketch the curves $y = e^x - 1$ and $y = 2e^{-x}$. **[3]**

(ii) Find the exact coordinates of the point of intersection of these curves. **[5]**

6 A curve is defined by the equation $(x + y)^2 = 4x$. The point $(1, 1)$ lies on this curve.

By differentiating implicitly, show that $\dfrac{dy}{dx} = \dfrac{2}{x + y} - 1$.

Hence verify that the curve has a stationary point at $(1, 1)$. **[4]**

7 Fig. 7 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = 1 + 2\arctan x$, $x \in \mathbb{R}$. The scales on the x- and y-axes are the same.

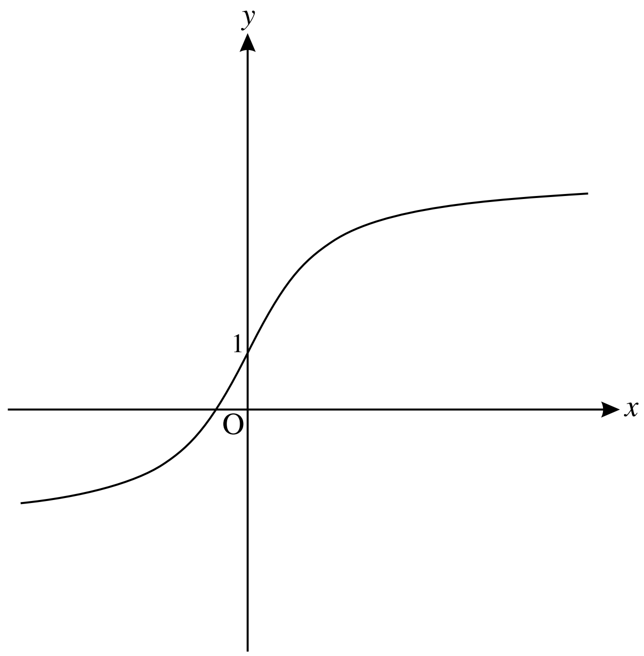

Fig. 7

(i) Find the range of f, giving your answer in terms of π. **[3]**

(ii) Find $\mathrm{f}^{-1}(x)$, and add a sketch of the curve $y = \mathrm{f}^{-1}(x)$ to the copy of Fig. 7. **[5]**

Section B (36 Marks)

8 **(i)** Use the substitution $u = 1 + x$ to show that

$$\int_0^1 \frac{x^3}{1+x}\,dx = \int_a^b \left(u^2 - 3u + 3 - \frac{1}{u}\right) du,$$

where a and b are to be found.

Hence evaluate $\displaystyle\int_0^1 \frac{x^3}{1+x}\,dx$, giving your answer in exact form. **[7]**

Fig. 8 shows the curve $y = x^2 \ln(1 + x)$.

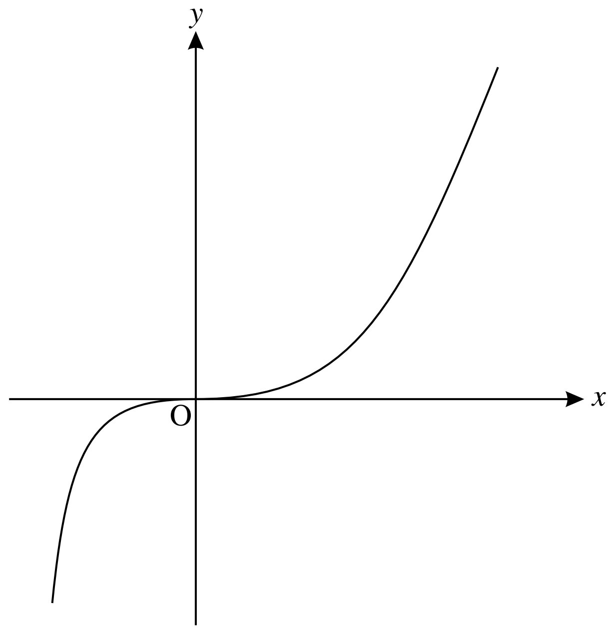

Fig. 8

(ii) Find $\dfrac{dy}{dx}$.

Verify that the origin is a stationary point of the curve. **[5]**

(iii) Using integration by parts, and the result of part **(i)**, find the exact area enclosed by the curve $y = x^2 \ln(1 + x)$, the x-axis and the line $x = 1$. **[6]**

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{\cos^2 x}$, $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$, together with its asymptotes $x = \frac{1}{2}\pi$ and $x = -\frac{1}{2}\pi$.

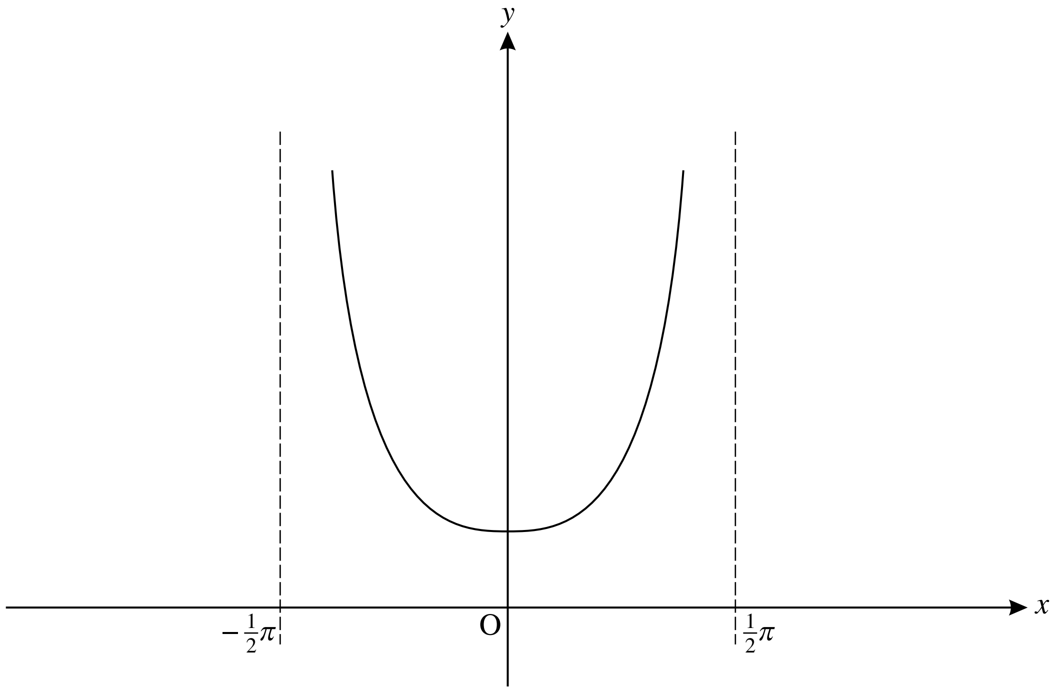

Fig. 9

(i) Use the quotient rule to show that the derivative of $\dfrac{\sin x}{\cos x}$ is $\dfrac{1}{\cos^2 x}$. **[3]**

(ii) Find the area bounded by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \frac{1}{4}\pi$. **[3]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \frac{1}{2}\mathrm{f}\left(x + \frac{1}{4}\pi\right)$.

(iii) Verify that the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ cross at $(0,\ 1)$. **[3]**

(iv) State a sequence of two transformations such that the curve $y = \mathrm{f}(x)$ is mapped to the curve $y = \mathrm{g}(x)$.

On the copy of Fig. 9, sketch the curve $y = \mathrm{g}(x)$, indicating clearly the coordinates of the minimum point and the equations of the asymptotes to the curve. **[8]**

(v) Use your result from part **(ii)** to write down the area bounded by the curve $y = \mathrm{g}(x)$, the x-axis, the y-axis and the line $x = -\frac{1}{4}\pi$. **[1]**

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ADVANCED GCE

MATHEMATICS (MEI) **4753/01**

Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
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- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 Solve the equation $|2x - 1| = |x|$. **[4]**

2 Given that $f(x) = 2\ln x$ and $g(x) = e^x$, find the composite function $gf(x)$, expressing your answer as simply as possible. **[3]**

3 **(i)** Differentiate $\dfrac{\ln x}{x^2}$, simplifying your answer. **[4]**

(ii) Using integration by parts, show that $\displaystyle\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x}(1 + \ln x) + c$. **[4]**

4 The height h metres of a tree after t years is modelled by the equation

$$h = a - be^{-kt},$$

where a, b and k are positive constants.

(i) Given that the long-term height of the tree is 10.5 metres, and the initial height is 0.5 metres, find the values of a and b. **[3]**

(ii) Given also that the tree grows to a height of 6 metres in 8 years, find the value of k, giving your answer correct to 2 decimal places. **[3]**

5 Given that $y = x^2\sqrt{1 + 4x}$, show that $\dfrac{dy}{dx} = \dfrac{2x(5x + 1)}{\sqrt{1 + 4x}}$. **[5]**

6 A curve is defined by the equation $\sin 2x + \cos y = \sqrt{3}$.

(i) Verify that the point $P\left(\frac{1}{6}\pi, \frac{1}{6}\pi\right)$ lies on the curve. **[1]**

(ii) Find $\dfrac{dy}{dx}$ in terms of x and y.

Hence find the gradient of the curve at the point P. **[5]**

7 **(i)** Multiply out $(3^n + 1)(3^n - 1)$. **[1]**

(ii) Hence prove that if n is a positive integer then $3^{2n} - 1$ is divisible by 8. **[3]**

Section B (36 marks)

8

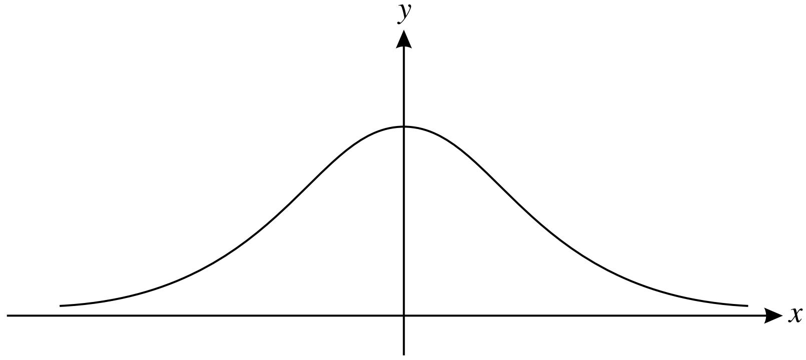

Fig. 8

Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{\mathrm{e}^x + \mathrm{e}^{-x} + 2}$.

(i) Show algebraically that $\mathrm{f}(x)$ is an even function, and state how this property relates to the curve $y = \mathrm{f}(x)$. **[3]**

(ii) Find $\mathrm{f}'(x)$. **[3]**

(iii) Show that $\mathrm{f}(x) = \dfrac{\mathrm{e}^x}{(\mathrm{e}^x + 1)^2}$. **[2]**

(iv) Hence, using the substitution $u = \mathrm{e}^x + 1$, or otherwise, find the exact area enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, and the lines $x = 0$ and $x = 1$. **[5]**

(v) Show that there is only one point of intersection of the curves $y = \mathrm{f}(x)$ and $y = \frac{1}{4}\mathrm{e}^x$, and find its coordinates. **[5]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$. The endpoints of the curve are P $(-\pi, 1)$ and Q $(\pi, 3)$, and $\mathrm{f}(x) = a + \sin bx$, where a and b are constants.

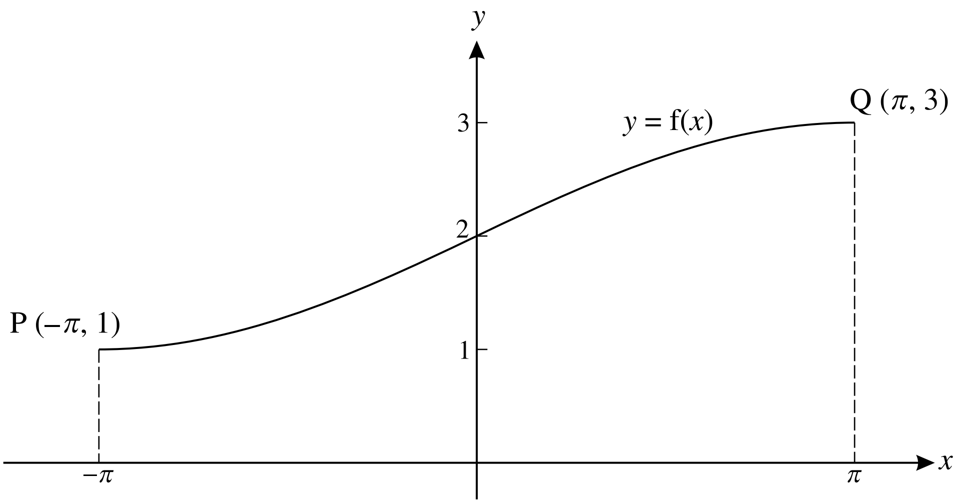

Fig. 9

(i) Using Fig. 9, show that $a = 2$ and $b = \frac{1}{2}$. **[3]**

(ii) Find the gradient of the curve $y = \mathrm{f}(x)$ at the point $(0, 2)$.

Show that there is no point on the curve at which the gradient is greater than this. **[5]**

(iii) Find $\mathrm{f}^{-1}(x)$, and state its domain and range.

Write down the gradient of $y = \mathrm{f}^{-1}(x)$ at the point $(2, 0)$. **[6]**

(iv) Find the area enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \pi$. **[4]**

RECOGNISING ACHIEVEMENT

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Friday 20 January 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

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- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Differentiate $x^2 \tan 2x$. **[3]**

2 The functions f(x) and g(x) are defined as follows.

$$\mathrm{f}(x) = \ln x, \qquad x > 0$$
$$\mathrm{g}(x) = 1 + x^2, \qquad x \in \mathbb{R}$$

Write down the functions fg(x) and gf(x), and state whether these functions are odd, even or neither. **[4]**

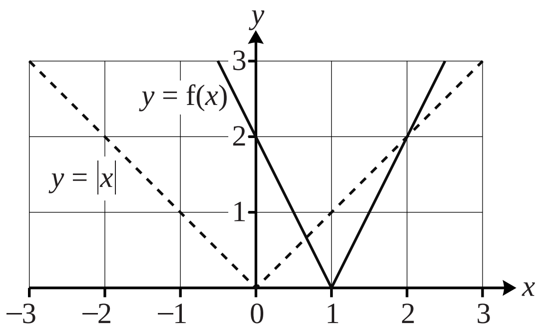
3 Show that $\displaystyle\int_0^{\frac{\pi}{2}} x \cos\tfrac{1}{2}x \, \mathrm{d}x = \frac{\sqrt{2}}{2}\pi + 2\sqrt{2} - 4$. **[5]**

4 Prove or disprove the following statement:

'No cube of an integer has 2 as its units digit.' **[2]**

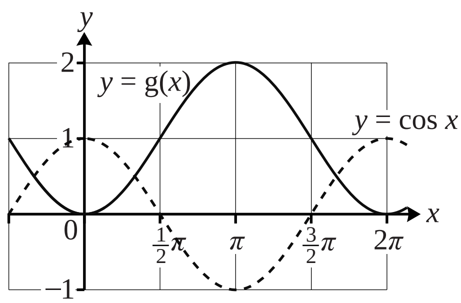
5 Each of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ below is obtained using a sequence of two transformations applied to the corresponding dashed graph. In each case, state suitable transformations, and hence find expressions for f(x) and g(x).

(i)



[3]

(ii)



[3]

6 Oil is leaking into the sea from a pipeline, creating a circular oil slick. The radius r metres of the oil slick t hours after the start of the leak is modelled by the equation

$$r = 20(1 - e^{-0.2t}).$$

(i) Find the radius of the slick when $t = 2$, and the rate at which the radius is increasing at this time. **[4]**

(ii) Find the rate at which the area of the slick is increasing when $t = 2$. **[4]**

7 Fig. 7 shows the curve $x^3 + y^3 = 3xy$. The point P is a turning point of the curve.

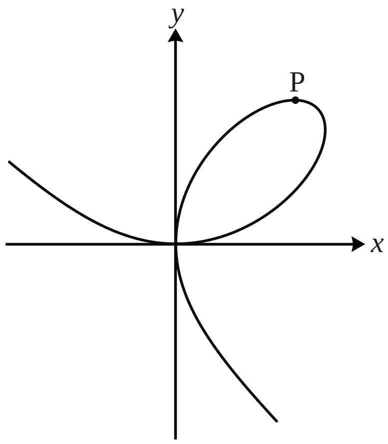


Fig. 7

(i) Show that $\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$. **[4]**

(ii) Hence find the exact x-coordinate of P. **[4]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = \frac{x}{\sqrt{x-2}}$, together with the lines $y = x$ and $x = 11$. The curve meets these lines at P and Q respectively. R is the point (11, 11).

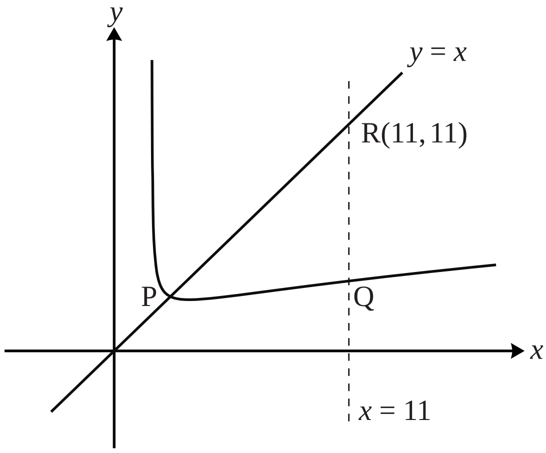


Fig. 8

(i) Verify that the x-coordinate of P is 3. **[2]**

(ii) Show that, for the curve, $\frac{dy}{dx} = \frac{x-4}{2(x-2)^{\frac{3}{2}}}$.

Hence find the gradient of the curve at P. Use the result to show that the curve is **not** symmetrical about $y = x$. **[7]**

(iii) Using the substitution $u = x - 2$, show that $\int_3^{11} \frac{x}{\sqrt{x-2}}\,dx = 25\frac{1}{3}$.

Hence find the area of the region PQR bounded by the curve and the lines $y = x$ and $x = 11$. **[9]**

9 Fig. 9 shows the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$. The function $y = \mathrm{f}(x)$ is given by

$$\mathrm{f}(x) = \ln\left(\frac{2x}{1+x}\right), \quad x > 0.$$

The curve $y = \mathrm{f}(x)$ crosses the x-axis at P, and the line $x = 2$ at Q.

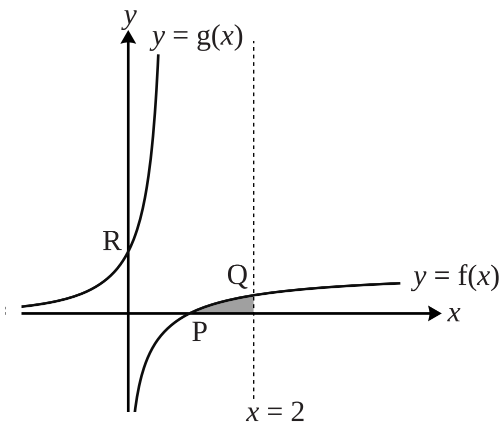


Fig. 9

(i) Verify that the x-coordinate of P is 1.

Find the exact y-coordinate of Q. **[2]**

(ii) Find the gradient of the curve at P. [Hint: use $\ln \frac{a}{b} = \ln a - \ln b$.] **[4]**

The function $\mathrm{g}(x)$ is given by

$$\mathrm{g}(x) = \frac{\mathrm{e}^x}{2 - \mathrm{e}^x}, \quad x < \ln 2.$$

The curve $y = \mathrm{g}(x)$ crosses the y-axis at the point R.

(iii) Show that $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

Write down the gradient of $y = \mathrm{g}(x)$ at R. **[5]**

(iv) Show, using the substitution $u = 2 - \mathrm{e}^x$ or otherwise, that $\int_0^{\ln \frac{4}{3}} \mathrm{g}(x)\,\mathrm{d}x = \ln \frac{3}{2}$.

Using this result, show that the exact area of the shaded region shown in Fig. 9 is $\ln \frac{32}{27}$.
[Hint: consider its reflection in $y = x$.] **[7]**

RECOGNISING ACHIEVEMENT

Thursday 21 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (36 marks)

1 Show that $\int_1^2 \frac{1}{\sqrt{3x-2}}\,dx = \frac{2}{3}$. **[5]**

2 Solve the inequality $|2x+1| > 4$. **[3]**

3 Find the gradient at the point (0, ln 2) on the curve with equation $e^{2y} = 5 - e^{-x}$. **[4]**

4 Fig. 4 shows the curve $y = f(x)$, where $f(x) = \sqrt{1-9x^2}$, $-a \leqslant x \leqslant a$.

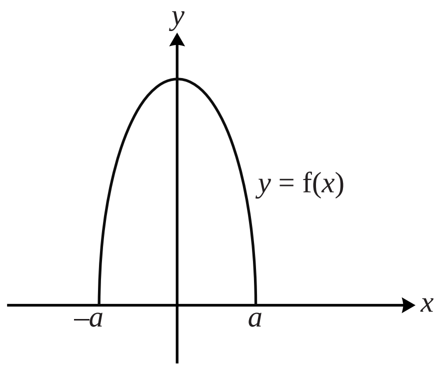

Fig. 4

(i) Find the value of a. **[2]**

(ii) Write down the range of $f(x)$. **[1]**

(iii) Sketch the curve $y = f(\frac{1}{3}x) - 1$. **[3]**

5 A termites' nest has a population of P million. P is modelled by the equation $P = 7 - 2e^{-kt}$, where t is in years, and k is a positive constant.

(i) Calculate the population when $t = 0$, and the long-term population, given by this model. **[3]**

(ii) Given that the population when $t = 1$ is estimated to be 5.5 million, calculate the value of k. **[3]**

6 Fig. 6 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = 2\arcsin x$, $-1 \leqslant x \leqslant 1$.

Fig. 6 also shows the curve $y = \mathrm{g}(x)$, where $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

P is the point on the curve $y = \mathrm{f}(x)$ with x-coordinate $\frac{1}{2}$.

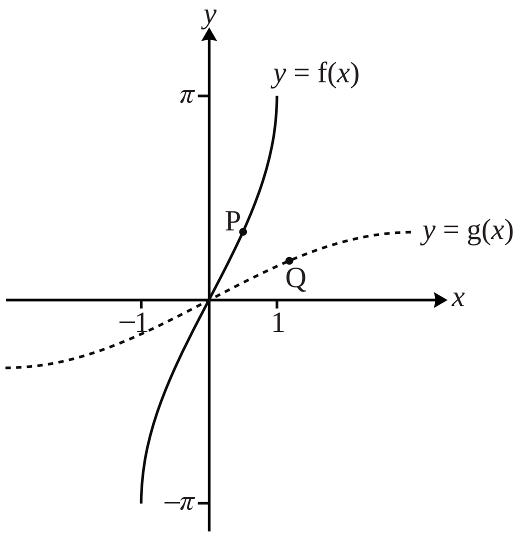

Fig. 6

(i) Find the y-coordinate of P, giving your answer in terms of π. **[2]**

The point Q is the reflection of P in $y = x$.

(ii) Find $\mathrm{g}(x)$ and its derivative $\mathrm{g}'(x)$. Hence determine the exact gradient of the curve $y = \mathrm{g}(x)$ at the point Q.

Write down the exact gradient of $y = \mathrm{f}(x)$ at the point P. **[6]**

7 You are given that $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are odd functions, defined for $x \in \mathbb{R}$.

(i) Given that $\mathrm{s}(x) = \mathrm{f}(x) + \mathrm{g}(x)$, prove that $\mathrm{s}(x)$ is an odd function. **[2]**

(ii) Given that $\mathrm{p}(x) = \mathrm{f}(x)\mathrm{g}(x)$, determine whether $\mathrm{p}(x)$ is odd, even or neither. **[2]**

Section B (36 marks)

8 Fig. 8 shows a sketch of part of the curve $y = x \sin 2x$, where x is in radians.

The curve crosses the x-axis at the point P. The tangent to the curve at P crosses the y-axis at Q.

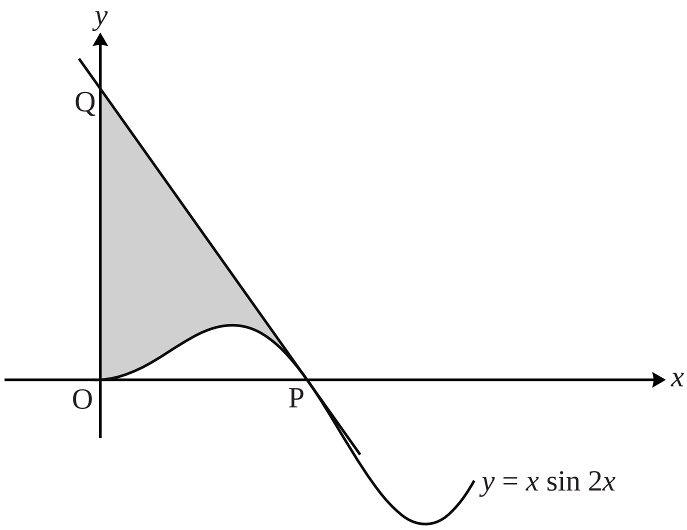

Fig. 8

(i) Find $\frac{dy}{dx}$. Hence show that the x-coordinates of the turning points of the curve satisfy the equation $\tan 2x + 2x = 0$. **[4]**

(ii) Find, in terms of π, the x-coordinate of the point P.

Show that the tangent PQ has equation $2\pi x + 2y = \pi^2$.

Find the exact coordinates of Q. **[7]**

(iii) Show that the exact value of the area shaded in Fig. 8 is $\frac{1}{8}\pi(\pi^2 - 2)$. **[7]**

9 Fig. 9 shows the curve $y = f(x)$, which has a y-intercept at P(0, 3), a minimum point at Q(1, 2), and an asymptote $x = -1$.

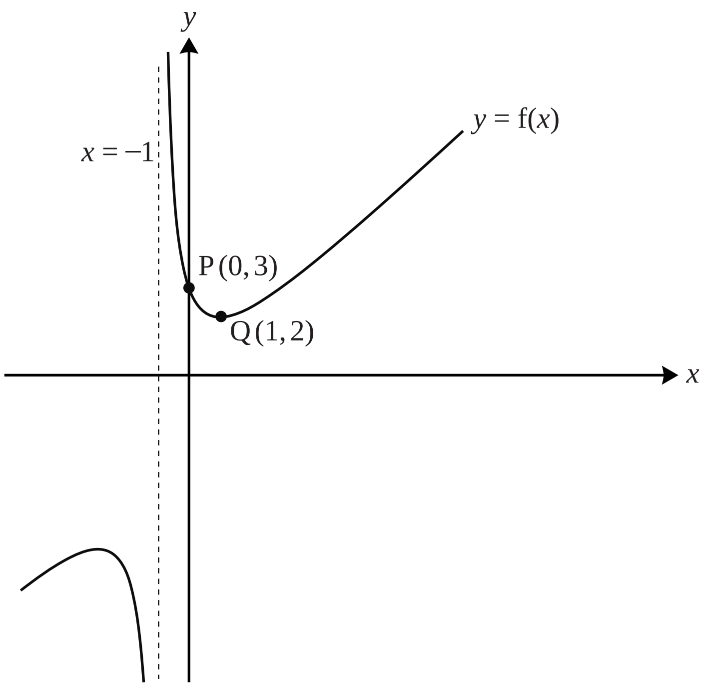

Fig. 9

(i) Find the coordinates of the images of the points P and Q when the curve $y = f(x)$ is transformed to

(A) $y = 2f(x)$,

(B) $y = f(x + 1) + 2$. **[4]**

You are now given that $f(x) = \dfrac{x^2 + 3}{x + 1}$, $x \neq -1$.

(ii) Find $f'(x)$, and hence find the coordinates of the other turning point on the curve $y = f(x)$. **[6]**

(iii) Show that $f(x - 1) = x - 2 + \dfrac{4}{x}$. **[3]**

(iv) Find $\displaystyle\int_a^b \left(x - 2 + \frac{4}{x}\right) dx$ in terms of a and b.

Hence, by choosing suitable values for a and b, find the exact area enclosed by the curve $y = f(x)$, the x-axis, the y-axis and the line $x = 1$. **[5]**

OCR
RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

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- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
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- Answer **all** the questions.
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- You are permitted to use a scientific or graphical calculator in this paper.
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- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (36 marks)

1 **(i)** Given that $y = e^{-x}\sin 2x$, find $\frac{dy}{dx}$. **[3]**

(ii) Hence show that the curve $y = e^{-x}\sin 2x$ has a stationary point when $x = \frac{1}{2}\arctan 2$. **[3]**

2 A curve has equation $x^2 + 2y^2 = 4x$.

(i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y. **[3]**

(ii) Hence find the exact coordinates of the stationary points of the curve. [You need not determine their nature.] **[3]**

3 Express $1 < x < 3$ in the form $|x - a| < b$, where a and b are to be determined. **[2]**

4 The temperature θ °C of water in a container after t minutes is modelled by the equation

$$\theta = a - be^{-kt},$$

where a, b and k are positive constants.

The initial and long-term temperatures of the water are 15 °C and 100 °C respectively. After 1 minute, the temperature is 30 °C.

(i) Find a, b and k. **[6]**

(ii) Find how long it takes for the temperature to reach 80 °C. **[2]**

5 The driving force F newtons and velocity v km s^{-1} of a car at time t seconds are related by the equation $F = \frac{25}{v}$.

(i) Find $\frac{dF}{dv}$. **[2]**

(ii) Find $\frac{dF}{dt}$ when $v = 50$ and $\frac{dv}{dt} = 1.5$. **[3]**

6 Evaluate $\int_0^3 x(x+1)^{-\frac{1}{2}}\,dx$, giving your answer as an exact fraction. **[5]**

7 **(i)** Disprove the following statement:

$$3^n + 2 \text{ is prime for all integers } n \geqslant 0.$$ **[2]**

(ii) Prove that no number of the form 3^n (where n is a positive integer) has 5 as its final digit. **[2]**

Section B (36 marks)

8 Fig. 8 shows parts of the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, where $\mathrm{f}(x) = \tan x$ and $\mathrm{g}(x) = 1 + \mathrm{f}(x - \frac{1}{4}\pi)$.

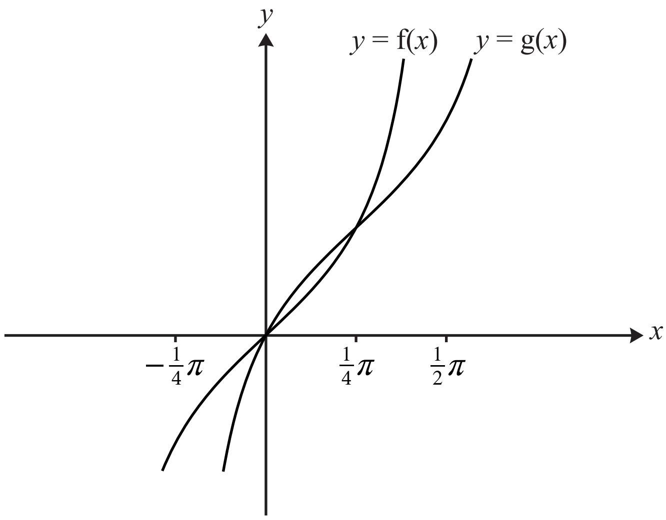

Fig. 8

(i) Describe a sequence of two transformations which maps the curve $y = \mathrm{f}(x)$ to the curve $y = \mathrm{g}(x)$. **[4]**

It can be shown that $\mathrm{g}(x) = \dfrac{2\sin x}{\sin x + \cos x}$.

(ii) Show that $\mathrm{g}'(x) = \dfrac{2}{(\sin x + \cos x)^2}$. Hence verify that the gradient of $y = \mathrm{g}(x)$ at the point $(\frac{1}{4}\pi, 1)$ is the same as that of $y = \mathrm{f}(x)$ at the origin. **[7]**

(iii) By writing $\tan x = \dfrac{\sin x}{\cos x}$ and using the substitution $u = \cos x$, show that $\displaystyle\int_0^{\frac{1}{4}\pi} \mathrm{f}(x)\,\mathrm{d}x = \int_{\frac{1}{\sqrt{2}}}^{1} \frac{1}{u}\,\mathrm{d}u$.

Evaluate this integral exactly. **[4]**

(iv) Hence find the exact area of the region enclosed by the curve $y = \mathrm{g}(x)$, the x-axis and the lines $x = \frac{1}{4}\pi$ and $x = \frac{1}{2}\pi$. **[2]**

9 Fig. 9 shows the line $y = x$ and the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{1}{2}(\mathrm{e}^x - 1)$. The line and the curve intersect at the origin and at the point P(a, a).

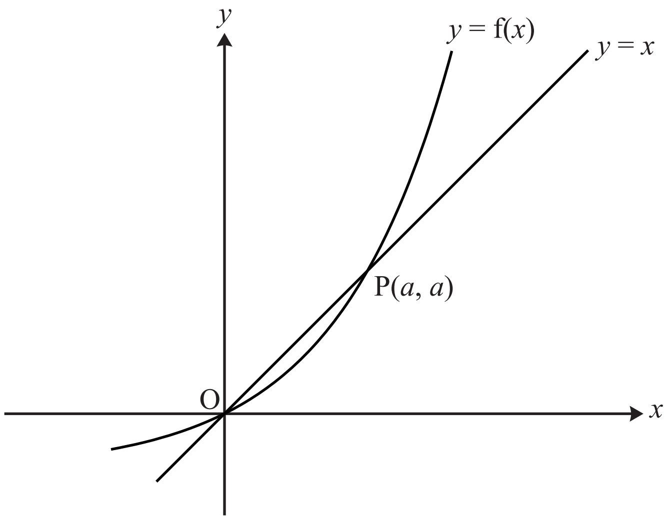

Fig. 9

(i) Show that $\mathrm{e}^a = 1 + 2a$. **[1]**

(ii) Show that the area of the region enclosed by the curve, the x-axis and the line $x = a$ is $\frac{1}{2}a$. Hence find, in terms of a, the area enclosed by the curve and the line $y = x$. **[6]**

(iii) Show that the inverse function of $\mathrm{f}(x)$ is $\mathrm{g}(x)$, where $\mathrm{g}(x) = \ln(1 + 2x)$. Add a sketch of $y = \mathrm{g}(x)$ to the copy of Fig. 9. **[5]**

(iv) Find the derivatives of $\mathrm{f}(x)$ and $\mathrm{g}(x)$. Hence verify that $\mathrm{g}'(a) = \dfrac{1}{\mathrm{f}'(a)}$.

Give a geometrical interpretation of this result. **[7]**

RECOGNISING ACHIEVEMENT

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (36 marks)

1 Fig. 1 shows the graphs of $y = |x|$ and $y = a|x + b|$, where a and b are constants. The intercepts of $y = a|x + b|$ with the x- and y-axes are $(-1, 0)$ and $\left(0, \frac{1}{2}\right)$ respectively.

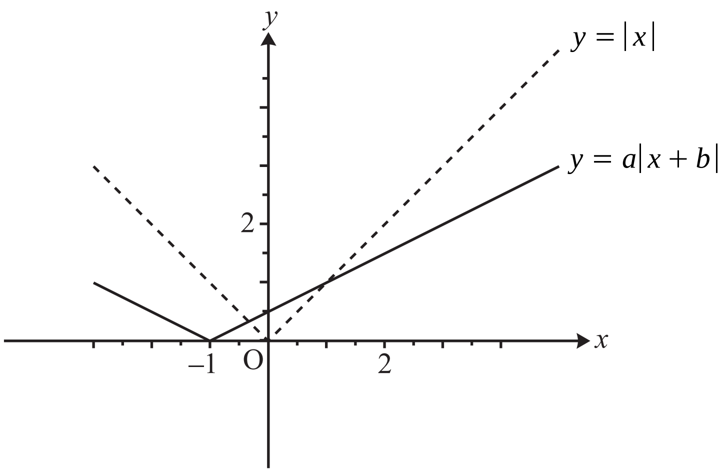

Fig. 1

(i) Find a and b. **[2]**

(ii) Find the coordinates of the two points of intersection of the graphs. **[4]**

2 **(i)** Factorise fully $n^3 - n$. **[2]**

(ii) Hence prove that, if n is an integer, $n^3 - n$ is divisible by 6. **[2]**

3 The function $f(x)$ is defined by $f(x) = 1 - 2\sin x$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$. Fig. 3 shows the curve $y = f(x)$.

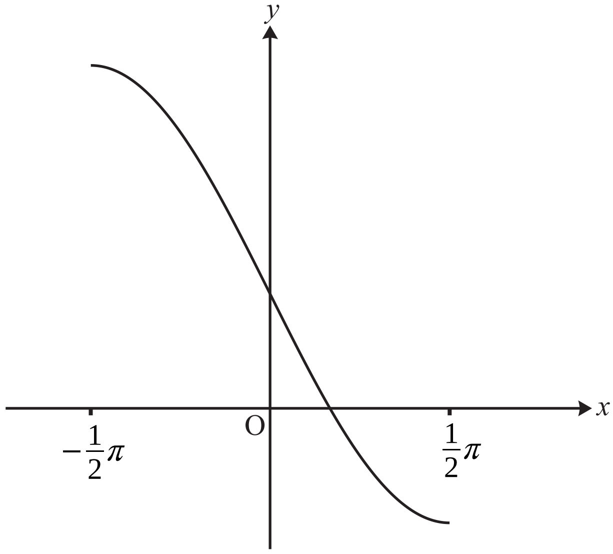

Fig. 3

(i) Write down the range of the function $f(x)$. **[2]**

(ii) Find the inverse function $f^{-1}(x)$. **[3]**

(iii) Find $f'(0)$. Hence write down the gradient of $y = f^{-1}(x)$ at the point $(1, 0)$. **[3]**

4 Water flows into a bowl at a constant rate of $10\,\text{cm}^3\,\text{s}^{-1}$ (see Fig. 4).

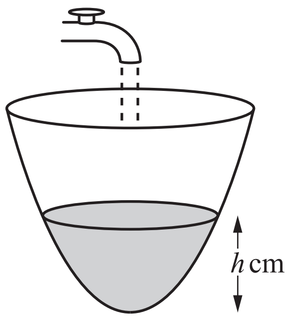

Fig. 4

When the depth of water in the bowl is h cm, the volume of water is V cm^3, where $V = \pi h^2$. Find the rate at which the depth is increasing at the instant in time when the depth is 5 cm. **[5]**

5 Given that $y = \ln\left(\sqrt{\dfrac{2x-1}{2x+1}}\right)$, show that $\dfrac{dy}{dx} = \dfrac{1}{2x-1} - \dfrac{1}{2x+1}$. **[4]**

6 Using a suitable substitution or otherwise, show that $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{3 + \cos 2x}\,dx = \frac{1}{2}\ln 2$. **[5]**

7 **(i)** Show algebraically that the function $f(x) = \dfrac{2x}{1-x^2}$ is odd. **[2]**

Fig. 7 shows the curve $y = f(x)$ for $0 \leqslant x \leqslant 4$, together with the asymptote $x = 1$.

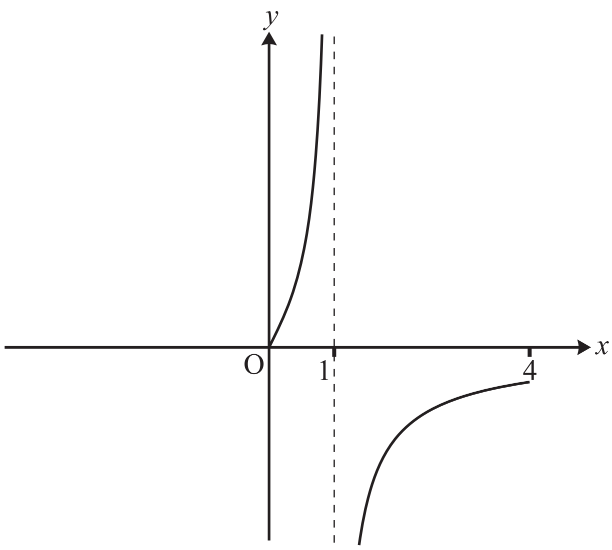

Fig. 7

(ii) Use the copy of Fig. 7 to complete the curve for $-4 \leqslant x \leqslant 4$. **[2]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (1-x)\mathrm{e}^{2x}$, with its turning point P.

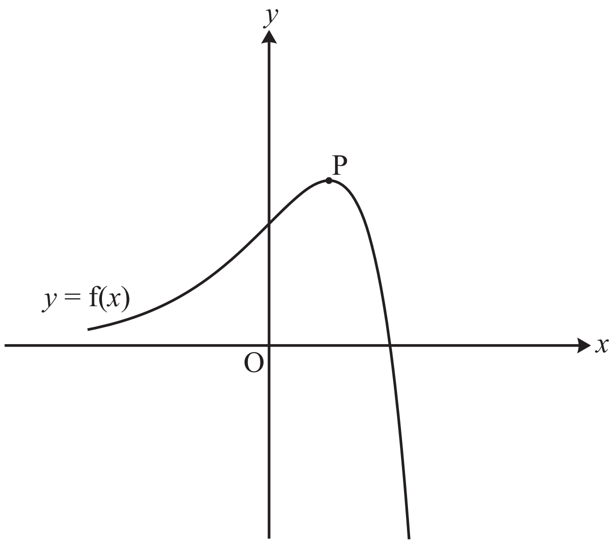

Fig. 8

(i) Write down the coordinates of the intercepts of $y = \mathrm{f}(x)$ with the x- and y-axes. **[2]**

(ii) Find the exact coordinates of the turning point P. **[6]**

(iii) Show that the exact area of the region enclosed by the curve and the x- and y-axes is $\frac{1}{4}(\mathrm{e}^2 - 3)$. **[5]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = 3\mathrm{f}\left(\frac{1}{2}x\right)$.

(iv) Express $\mathrm{g}(x)$ in terms of x.

Sketch the curve $y = \mathrm{g}(x)$ on the copy of Fig. 8, indicating the coordinates of its intercepts with the x- and y-axes and of its turning point. **[4]**

(v) Write down the exact area of the region enclosed by the curve $y = \mathrm{g}(x)$ and the x- and y-axes. **[1]**

9 Fig. 9 shows the curve with equation $y^3 = \dfrac{x^3}{2x-1}$. It has an asymptote $x = a$ and turning point P.

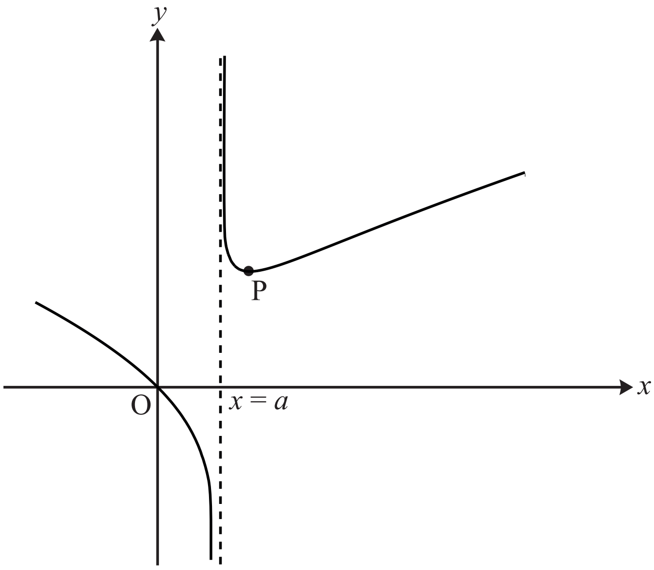

Fig. 9

(i) Write down the value of a. **[1]**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{4x^3 - 3x^2}{3y^2(2x-1)^2}$.

Hence find the coordinates of the turning point P, giving the y-coordinate to 3 significant figures. **[9]**

(iii) Show that the substitution $u = 2x - 1$ transforms $\displaystyle\int \frac{x}{\sqrt[3]{2x-1}}\,dx$ to $\displaystyle\frac{1}{4}\int \left(u^{\frac{2}{3}} + u^{-\frac{1}{3}}\right) du$.

Hence find the exact area of the region enclosed by the curve $y^3 = \dfrac{x^3}{2x-1}$, the x-axis and the lines $x = 1$ and $x = 4.5$. **[8]**

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